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REGULAR COTOTAL DOMINATION IN LINE GRAPH

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ABSTRACT:

A dominating set $D \subseteq V[L(G)]$ is called regular cototal dominating set, if the induced subgraph $\langle V[L(G)] - D\rangle$ is regular and has no isolated vertices. The minimum cardinality of vertices in a such a set is called regular cototal domination number in $L(G)$ and is denoted by $\gamma_{rct}[L(G)]$.

In this paper, we study some theoretic properties of $\gamma_{rct}[L(G)]$ and many bounds were obtained in terms of elements of G also relationship with other domination parameters were found.

KEY WORDS: Graph, Line graph , Regular cototal dominating set, Regular cototal domination number.

Subject classification number: AMS05C69, 05C70.

INTRODUCTION: In this paper, generally we follow the notations of Harary[3]. All the graphs considered here are finite and simple.

A line graph $L(G)$ is the graph whose vertices corresponds to the edges of G and two vertices in $L(G)$ are adjacent if and only if the corresponding edges in G are adjacent.

We begin by recalling some standard definitions from domination theory. A set $D \subseteq V[G]$ is called dominating set of G, if every vertex in $[V(G) - D]$ is adjacent to some vertex in D. The smallest cardinality of vertices in such a set is called the domination number of G and is denoted by $\gamma(G)$, see[5].

A dominating set $F \subseteq V[G]$ is said to be a total dominating set, if the induced subgraph $\langle F\rangle$ has no isolated vertices. The total domination number $\gamma_t(G)$ is the smallest cardinality of a total dominating set of G, see[2].

A dominating set $D \subseteq V(G)$ is a cototal dominating set if the induced subgraph $\langle V(G) - D\rangle$ has no isolated vertices. The cototal domination number $\gamma_{cot}(G)$ of G is the minimum cardinality of cototal domination set of G,see[6].

Analogously, a dominating set D of a line graph $L[G]$ is a regular cototal dominating set, if the induced subgraph $\langle V[L(G)] - D\rangle$ has no isolated vertices and regular. The regular cototal domination number $\gamma_{rct}[L(G)]$ is the minimum cardinality of a regular cototal dominating set of $L(G)$.

We initiate the study of regular cototal domination in line graph.

RESULTS:

In the following theorem,we give the regular cototal domination number of $L(G)$ for some standard graphs, which are straight forward.

Theorem1:

i) For any star $G = K_{1,n}$ $n \geq 3$ $\gamma_{rct}(L(K_{1.n})) = 1$.

ii) For any path $G = P_{3n+2}, n = 1,2,3 \dots \dots \dots, G = P_{3n+2}, n = 1,2,3 \dots \dots \dots, \gamma_{rct}\left(L(P_{3n+2})\right) = n + 1$.

$$\gamma_{rct}\left(L(P_n)\right) = n - 2\left\lfloor \frac{n}{3} \right\rfloor + 1 \text{ Except } G = P_{3n+2}.$$

iii) For any cycle $G = C_{3n}, \gamma_{rct}\left(L(C_{3n})\right) = n$

$$= n - 2\left\lfloor \frac{n}{3} \right\rfloor + 1, n = 1,2,3 \dots \dots \text{otherwise}$$

iv) For any Wheel $G = W_n$ $\quad \gamma_{rct}\left(L(W_n)\right) = n - 1, n = 4,5,6 \dots \dots \dots \dots$

v) For any bipartite graph $G = K_{m,n}$, $\quad \gamma_{rct}\left(L(K_{m,n})\right) = \min(m.n)$.

The following theorem relates the regular cototal domination in line graph and domination number in G in terms of number of edges of G and maximum degree of G.

Theorem2: For any connected (p,q) graph G, with $p \geq 3$ vertices $\gamma_{rct}\left(L(G)\right) + \gamma(G) \leq q + \Delta(G) - 1$.

Proof: Suppose $F = \{v_1, v_2, v_3, \dots \dots \dots, v_n\} \subseteq V(G)$ be the set of all non end vertices in G. Assume $A \subseteq F$ which covers all the vertices in G and $N[A] = V(G)$. Then clearly A forms a minimal dominating set of G. Now since $E(G) = V[L(G)]$ and let $B = \{e_1, e_2, e_3, \dots \dots \dots., e_n\}$ be the set of all edges which are incident to the vertices of A. Now without loss of generality, $\subseteq V[L(G)]$. Suppose $K = \{v_1, v_2, v_3, \dots \dots \dots, v_m\}$ be the set of all end vertices in $L(G)$, then $C = K \cup B_1$, where $B_1 \subseteq B$ forms a minimal cototal dominating set of $L(G)$.If the induced subgraph $\langle V[L(G)] - C \rangle$ has no isolated vertices and regular, then C forms a regular cototal dominating set of $L(G)$. Since $|E(G)| = q$ and for any connected graph G, then there exists atleast one vertex $v \in V(G)$ of maximum degree $\Delta(G)$. It follows that $|C| + |A| \leq |E(G)| + \Delta(G) - 1$ which gives $\gamma_{rct}\big(L(G)\big) + \gamma(G) \leq q + \Delta(G) - 1$.

A dominating set $D \subseteq E(G)$ is an edge dominating set of G, if every edge in $(E(G) - D)$ is adjacent to some edge in D. The edge domination number $\gamma'(G)$ of G is the minimum cardinality of edge dominating set of G, see[7].

In the following theorem, we develop a relationship between $\gamma_{rct}\big(L(G)\big)$ and $\gamma'(G)$ with number of vertices of G.

Theorem3: For any connected $(p.q)$ graph G, $\gamma_{rct}\big(L(G)\big) + \gamma'(G) \leq p + 1\ with\ G \neq K_p, p \geq 5$.

Proof: Suppose $G = K_p$. Then $\gamma_{rct}\big(L(G)\big) + \gamma'(G) \nleq p + 1$. Hence $G \neq K_p, p \geq$ 5 vertices. Let $E_1 = \{e_1, e_2, e_3, \ldots\ldots\ldots., e_n\} \subseteq E(G)$ be the minimal set of edges which covers all the edges in G, such that $N[E_1] = E(G)$. Then E_1 forms a minimal edge dominating set of G. Now since $E(G) = V[L(G)]$ and suppose $M_1 \subseteq E_1$ and $M_2 \subseteq E(G) - E_1$. Then in $L(G)$, $\{M_1 \cup M_2\} \subset V[L(G)]$. Now assume every vertex $v_i \in V[L(G)] - \{M_1 \cup M_2\}$ is adjacent to atleast one vertex of $v_j \in \{M_1 \cup M_2\}$. If the induced subgraph $\langle V[L(G) - \{M_1 \cup M_2\}\rangle$ has no isolated vertex and regular, then $\{M_1 \cup M_2\}$ forms a minimal regular cototal dominating set of $L(G)$. Since $|V(G)| = p$ such that $|M_1 \cup M_2| + |E_1| \leq |V(G)| + 1$ which gives $\gamma_{rct}\big(L(G)\big) + \gamma'(G) \leq p + 1$.

A dominating set $D \subseteq V(G)$ is called weak dominating set, if for every vertex $u \in V - D$ and there exists a vertex $v \in D$ with $\deg(v) \leq \deg(u)$ and u is adjacent to v. The weak domination number ,$\gamma_w(G)$ is the minimum cardinality of a minimal weak dominating set of G, see[12].

A dominating set $F \subseteq V(G)$ is called strong dominating set, if for every vertex $u \in V - F$ and there exists a vertex $v \in F$ with $\deg(v) \geq \deg(u)$ and u is adjacent to v. The strong domination number $\gamma_{st}(G)$ is the minimum cardinality of a minimal strong dominating set of G, see[12].

The following theorem estabilish a relationship between $\gamma_w(G), \gamma_{st}(G)\ and\ \alpha_1(G) with\ \gamma_{rct}\big(L(G)\big)$.

Theorem4: For any connected graph G, $\gamma_{rct}\big(L(G)\big)+\alpha_1(G)\geq\gamma_w(G)+\gamma_{st}(G)$ with $G\neq K_{3,3}-C_4$.

Proof: Let $A=\{e_1,e_2,e_3,\dots\dots\dots,e_n\}$ be the set of all end edges. Suppose $A_1=\{e_1,e_2,e_3,\dots\dots\dots,e_k\}\subseteq E(G)-A$. Then $A\cup C$, where $C\subseteq A_1$, be the minimal set of edges which covers all the vertices in G such that $|A\cup C|=\alpha_1(G)$. Suppose $F\subset V(G)$ be the minimal dominating set of G. If for every $v_i\in V-F$ is adjacent to atleast one vertex of $v_j\in F$ with $\deg(v_j)\geq\deg(v_i)$ and v_j is adjacent to v_i. Then F is a minimal strong dominating set of G. Further suppose there exists a vertex set $D\subseteq V(G)$ such that $\forall\, v_k\in D$ is adjacent to atleast one vertex of $V(G)-D$ and $N[D]=V(G)$. Clearly D is a minimal dominating set of G. Further if $\forall v_r\in V(G)-D, \deg(v_k)\leq\deg(v_r)\, and v_k\, is$ adjacent to v_r. Then D is a weak dominating set of G. Let $E_1=\{e_1,e_2,e_3,\dots\dots\dots,e_k\}\subseteq E(G)$ and $E_2=\{e_1,e_2,e_3,\dots\dots\dots,e_l\}\subseteq E(G)$. Then $e_i\in E_1\, are$ incident with $\forall v_i\in\gamma_{st} set$ of G and $e_j\in E_2\, are$ incident with $\forall v_j\in\gamma_w set$ of G. Let $E_3=\{e_1,e_2,e_3,\dots\dots\dots,e_k\}=E(G)$. Then $\{\ u_1,u_2,u_3\dots\dots u_n\}=V[L(G)]$ corresponding to the elements of E_3. Also $H_1=\{\ u_1,u_2,u_3\dots\dots u_k\}\subset V[L(G)]$ corresponding to the elements of E_1 and $H_2=\{\ u_1,u_2,u_3\dots\dots u_l\}\subset V[L(G)]$ corresponding to the elements of E_2. Suppose $S\subset V[L(G)] be$ the minimal set of vertices which covers all the vertices in $L(G)$ with $N[S]=V[L(G)]$. If the induced subgraph $\langle V[L(G)]-S\rangle$ does not contain any isolated vertex and regular, then S itself is a γ_{rct} −set of $L(G)$. $But\ it\ is$ verify that $|S|$+$|A\cup C|\geq|H_1|$+$|H_2|$ which gives $\gamma_{rct}\big(L(G)\big)+\alpha_1(G)\geq\gamma_w(G)+\gamma_{st}(G)$.

The following theorem relates total domination number of G and $\beta_0(G)$ with $\gamma_{rct}(L(G)]$.

Theorem5: For any connected (p,q) graph G, $\gamma_{rct}(L(G)]\leq\gamma_t+\beta_0(G)$.

Proof: Let $K=\{v_1,v_2,v_3,\dots\dots\dots,v_n\}\subseteq V(G)$ be the maximum set of vertices such that $dist(u,v)\geq 2$ and $N(u)\cap N(v)=x,\forall\, u,v\in K$ and $x\in V(G)-K$. Clearly $|K|=\beta_0(G)$. Let $A=\{v_1,v_2,v_3,\dots\dots\dots,v_k\}\subseteq V(G)$ be the minimal dominating set of G. If the induced subgraph $\langle A\rangle$ has no isolated vertices, then A forms a minimal $\gamma_t-set$of G. Otherwise, if $\deg(v_i)<1,\forall\, v_i\in A$, then attach the minimum number of vertices $\{u_i\}\in N(A)$ such that $A\cup\{u_i\}$ forms a minimal total dominating set of G. Let $D=\{v_1,v_2,v_3,\dots\dots\dots,v_m\}\subseteq V_1\big(L(G)\big)=E_1(G)$, where $E_1(G)$ is the set of edges which are incident with the vertices of $A\cup\{u_i\}$. suppose $\forall u_i\in V[L(G)]-D$ is adjacent to atleast one vertex of D such that $N[D]=V[L(G)]$. If the induced subgraph $\langle V[L(G)-D\rangle$ has no isolated vertex and regular, then D is a cototal dominating set of $L(G)$. Hence it follows that $|D|\leq|A\cup\{u_i\}|+|K|$ which gives $\gamma_{rct}(L(G)]\leq\gamma_t+\beta_0(G)$.

A function $f\colon V\rightarrow\{0,1,2\}$ satisfying the condition that every vertex u for which $f(u)=0$ is adjacent to atleast one vertex v for which $f(v)=2$ in G. The weight of Roman dominating function is the value $f(v) = \sum_{u \in v} f(u)$. The minimum weight of Roman dominating function on a graph G is called Roman dominating number of G and is denoted by γ_R(G).

A total dominating set F of a graph $G = (V, E)$ is coregular total dominating set, if the induced subgraph $\langle V - F \rangle$ is regular. The coregular total domination number $\gamma_{crt}(G)$ of G is the minimum cardinality of a coregular total dominating set of G, see[11].

Theorem6: For any connected $(p.q)$ graph $G, \gamma_{rct}(L(G)] + \gamma_{crt}(G) \geq \gamma_R + 1, G \neq \left(K_{3,3} - C_4\right)$.

Proof: Suppose the function $f: V \to \{0,1,2\}$ and $\{V_0, V_1, V_2\}$ is a partition of a vertex set $V(G)$ which is induced by f with $|V_i| = n_i$ for $i = 0,1,2$. Suppose V_2 dominates V_0, then $F = V_1 \cup V_2$ forms a minimal Roman dominating set of G. Suppose $D = \{v_1, v_2, v_3, \dots \dots \dots, v_n\} \subseteq V(G)$ be the minimal set of vertices which covers all the vertices in G and the induced subgraph $\langle D \rangle$ has no isolated vertex then D itself is a γ_t −set of G. Suppose the subgraph $D_1 = [V(G) - D]$ and $\langle D_1 \rangle$ is regular. Then D_1 forms a $\gamma_{rct}(G)$ set of G. Now since $V[L(G)] = E(G)$, let $B = \{e_1, e_2, e_3, \dots \dots \dots., e_k\}$ be the set of all edges which are incident to the vertices of D_1 . Now without loss of generality, $B \subseteq V[L(G)]$. Suppose $I = \{v_1, v_2, v_3, \dots \dots \dots, v_m\}$ be the set of all end vertices in $L(G)$, then $F_1 = I \cup B_1$ where $B_1 \subseteq B$ forms a minimal cototal dominating set of $L(G)$. If the induced subgraph $\langle V[L(G) - F_1 \rangle$ has no isolated vertex and regular, then F_1 forms a minimal regular cototal dominating set of $L(G)$. Since $\subseteq F_1$, then it follows that $|F_1| + |D_1| \geq |F| + 1$ which gives $\gamma_{rct}(L(G)] + \gamma_{crt}(G) \geq \gamma_R + 1, G \neq \left(K_{3,3} - C_4\right)$.

A dominating set $D \subseteq V(G)$ is called coregular split dominating set if the induced subgraph $\langle V(G) - D \rangle$ is regular and disconnected. The minimum cardinality of such a set is called a coregular split domination number and is denoted by $\gamma_{crs}(G)$, see[10].

A dominating set F of a graph $G = (V, E)$ is a coregular dominating set if the induced subgraph $\langle V - F \rangle$ is regular. The coregular domination number $\gamma_{cr}(G)$ of G is the minimum cardinality a coregular dominating set of G, see[9].

The following theorem gives relationship between $\gamma_{cr}(G), \gamma_{crs}(G)$ with$\gamma_{rct}(L(G)]$.

Theorem7: For any connected (p, q) graph $G, \gamma_{rct}(L(G)] + 1 \leq \gamma_{cr}(G)$ +$\gamma_{crs}(G)$, $G \neq W_p$

Proof: Suppose $G = W_p$. Then $\gamma_{rct}(L(G)] + 1 \nleq \gamma_{cr}(G)$ +$\gamma_{crs}(G)$. Hence $G \neq W_p$. Now suppose $D = \{v_1, v_2, v_3, \dots \dots \dots, v_n\} \subseteq V(G)$ is a dominating set of G such that the subgraph $D_1 = [V(G) - D]$ and the induced subgraph $\langle D_1 \rangle$ is regular then $|D_1| = \gamma_{cr}(G)$.Further let $S = \{v_1, v_2, v_3, \dots \dots \dots, v_k\} \subseteq V(G)$ be the minimal dominating set of G. Suppose the induced subgraph $\langle V - S \rangle$ is disconnected and regular, then S itself is a coregular split dominating set of G. Now if $M = \{e_1, e_2, e_3, \dots \dots \dots., e_n\}$ be the set of all edges which are incident to the vertices of D . Since $V[L(G)] = E(G)$ and $M \subseteq V[L(G)]$. Suppose $I = \{u_1, u_2, u_3, \dots \dots \dots, u_m\}$ be the set of all end vertices in $L(G)$.Then $N = M_1 \cup I$, where $M_1 \subseteq M$ and the induced subgraph $\langle V[L(G)] - N \rangle$ has no isolated vertices and regular. Then N forms a regular cototal dominating set of $L(G)$. Since $S \subset N$ the it follows that $|N| + 1 \leq |D_1| + S$, which gives $\gamma_{rct}(L(G)] + 1 \leq \gamma_{cr}(G) + \gamma_{crs}(G)$.

A dominating set $F \subseteq V(G)$ is a double dominating set of G, if each vertex in V is a dominated by atleast two vertices in F. The double domination number $\gamma_{dd}(G)$ of G is the minimum cardinality of a double dominating set of G, see[4].

Theorem8: For any connected (p, q) graph G with $p \geq 3$ vertices $\gamma_{rct}\big(L(G)\big) + diam(G) + \Delta'(G) \geq \gamma_{dd}(G) + \alpha_{0.} G \neq C_n, n > 5$.

Proof: Suppose $G = C_n, n > 5$. Then $\gamma_{rct}\big(L(G)\big) + diam(G) + \Delta'(G) < \gamma_{dd}(G) + \alpha_{0.}$ Hence $G \neq C_n, n > 5$. Let $J = \{e_1, e_2, e_3, \dots \dots \dots., e_n\} \subseteq E(G)$ be the minimal set of edges which constitute the longest path between any two distinct vertices $s, t \in V(G)$ such that $dist(s, t) = diam(G)$. Further let $S = \{v_1, v_2, v_3, \dots \dots \dots, v_n\} \subseteq V(G)$ be the set of vertices with $\deg(v_i) \geq 2, \forall v_i \in S, 1 \leq i \leq n$ covers all the edges in G. Clearly $|S| = \alpha_0(G)$. Suppose $D = \{v_1, v_2, v_3, \dots \dots \dots, v_m\} \subseteq V(G)$ be the minimal dominating set of G. Now consider $M = V(G) - D$ and $= \{v_1, v_2, v_3, \dots \dots \dots, v_k\} \subseteq M$. Then $D^d = D \cup N$ forms a double dominating set of G. Now since $E(G) = V[L(G)]$ and $F = \{e_1, e_2, e_3, \dots \dots \dots., e_m\}$ be the set of all edges which are incident to the vertices of D^d. Now without loss of generality $F \subseteq V[L(G)]$. Suppose $B = \{v_1, v_2, v_3, \dots \dots \dots, v_i\}$ be the set of all end vertices in $L(G)$. Then $C = B \cup A_1$ where $A_1 \subseteq F$ forms a minimal cototal dominating set of $L(G)$. If the induced subgraph $\langle V[L(G)] - C \rangle$ has no isolated vertices and regular, then C forms a regular cototal dominating set of $L(G)$. Since there exists atleast one edge $e \in E(G)$ of maximum degree, then $\Delta'(G) = \deg(e)$. It follows that $|C| + |dist(s, t)| + |\deg(e)| \geq |D^d| + |S|$ which gives $\gamma_{rct}\big(L(G)\big) + diam(G) + \Delta'(G) \geq \gamma_{dd}(G) + \alpha_{0.}$

A dominating set $D \subseteq V(G)$ of graph $G$is a connected dominating set, if the induced subgraph $\langle D \rangle$ is connected. The connected domination number $\gamma_c(G)$ of G is the smallest cardinality of a connected dominating set of G, see[13].

Theorem9: For any connected (p, q) graph G with $p \geq 3$ vertices $\gamma_{rct}\big(L(G)\big) + \beta_1 \geq \gamma_c$.

Proof: Let $D \subseteq V(G)$ be a minimal dominating set of G. If the induced subgraph $\langle D \rangle$ has exactly one component, then D is a connected dominating set of G. Otherwise consider $v_i \in V - D$ such that $D \cup \{v_i\}$ is connected . Hence $(D \cup \{v_i\})$ is connected dominating set of G. Let $A = \{e_1, e_2, e_3, \dots \dots \dots., e_n\} \subseteq E(G)$ be the maximal set of edges such that $N(e_i) \cap N\big(e_j\big) = e$, For every $e_i, e_j \in A, 1 \leq i \leq n, 1 \leq j \leq n$ and $e \in E(G) - A$. Clearly A forms a maximal edge independent set in G. Since $E(G) = V[L(G)]$ and suppose $B \subseteq A$ and $C =$

$E(G)-A$. Then in $L(G)$, $\{B\cup C\}\subset V[L(G)]$. Now assume every vertex $\{v_j\}\in V[L(G)]-\{B\cup C\}$ is adjacent to atleast one vertex of $v_k\in\{B\cup C\}$. If the induced subgraph $\langle V[L(G)]-\{B\cup C\}\rangle$ has no isolated vertex and regular, then $(B\cup C)$ forms a minimal regular cototal dominating set of $L(G)$. Hence $|B\cup C|+|A|\geq|D\cup\{v_i\}|$ which gives $\gamma_{rct}(L(G))+\beta_1\geq\gamma_c$.

A perfect dominating set $D\subseteq V(G)$ is called regular perfect dominating set, if the induced subgraph $\langle D\rangle$ is regular. The minimum cardinality of D is called a regular perfect domination number in a graph G and is denoted by $\gamma_{rp}(G), see$[8].

A dominating set $F\subseteq V(G)$ is an independent dominating set, if the induced subgraph $\langle F\rangle$ has no edges. The independent domination number $i(G)$ of G is the minimum cardinality of independent dominating set, see[1].

In the following theorem, we develop a relation ship between $i(G),\gamma_{rp}(G),\gamma_s(G) with\ \gamma_{rct}(L(G))$.

Theorem10: For any connected (p,q) graph G, $\gamma_{rct}(L(G))+\gamma_s(G)\geq i(G)+\gamma_{rp}(G), G\neq C_n, where\ n\ is\ prime\ number\geq 5$.

Proof: Suppose $G=C_n(n\ is\ prime\ number\geq 5)$.. Then $\gamma_{rct}(L(G))+\gamma_s(G)\ngeq i(G)+\gamma_{rp}(G)$ a contradiction . Hence $G\neq C_n(n=5,7,11,\dots\dots)$. Suppose $D=\{v_1,v_2,v_3,\dots\dots\dots,v_n\}\subseteq V(G)$ be the minimal dominating set of G. If $\forall\, v_i\in D, \deg(v_i)=0$, then D is independent dominating set of G. Further let, $F=\{v_1,v_2,v_3,\dots\dots\dots,v_m\}\subseteq V(G)$ and $N[F]=V(G)$ such that $\forall\, v_j\in V(G)-F$ is adjacent to exactly one vertex of F and if $|N(u)\cap F|=1$ for each $u\in V-F$. Then $F\ is$ the minimal perfect dominating set of $V(G)$. If the induced subgraph $\langle F\rangle$ is regular, then F is a regular perfect dominating set of G. Let $M=\{v_1,v_2,v_3,\dots\dots\dots,v_k\}\subseteq V(G)$ be the minimal dominating set of G. Suppose induced subgraph $\langle V-M\rangle$ is disconnected. Then M itself is a split dominating set of G. Let $K=\{u_1,u_2,u_3\dots\dots\dots,u_n\}\subseteq V[L(G)]=E_1$(G) where E_1(G) is the set of edges which are incident with the vertices of F. Suppose $\forall\, u_i\in V[L(G)]-K$ is adjacent to atleast one vertex of $K\ such\ that\ N[K]=V[L(G)] and$ if the induced subgraph $\langle V[L(G)]-K\rangle$ does not contain isolated vertex and regular, then K is a $\gamma_{rct}(L(G))$ of $L(G). Since\ D\subseteq K, M\subseteq K$ for any connected graph G, then we have $|K|+|M|\geq|D|+|F|\ which\ gives\ \gamma_{rct}(L(G))+\gamma_s(G)\geq i(G)+\gamma_{rp}(G)$.

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