Some Anisotropic Spherically Symmetric Static Solutions Filled with Ferro Fluid

By :

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ABSTRACT :

The present paper provides some static solutions of Einstein-Maxwell's equations for anistropic ferrowfluid using spherically symmetric metric under different specific conditions. Various physical and geometric features have been found and discussed.

Key Words : Anisotropic, ferrowfluid, density, spherical symmetry, metric.

1. Introduction

A pretty number of researchers have found their interest in generating solutions of Einstein-Maxwell field equations which are very useful and relevant in general relativity [8, 9, 13]. These solutions provide deep knowledge of space-time and furthermore these are not connected to a specified choice of parameters and basic conditions. The latest survey of isotropic solutions of Einstein's field equations for spherical symmetry was given by Krammer et. al. [9]. Further Yodzis et. al. [17] presented a solution which provides naked singularities in the spherical gravitational collapse of anisotropic matter. Bowers and Liang [4] have investigated and discussed anisotropic spheres with useful impacts in astrophysics. A useful technique to obtain interior solutions of Einstein's equations for anisotropic matter from known solutions of such matter was established by Cosenza et. al. [6]. Stewart [16] has presented a large class of anisotropic interior solutions for static conformally flat spherically symmetric metric with a feedom to choose the functional form of mass distribution. A comparision of properties between isotropc and anisotropic spheres have been evolved by Cosenza et. al. [7] and Bayin [2]. Punce de Leon [13, 14] has provided various solutions applying new methods. Berman [3] has found an anisotropc cosmological solution considering Bianchi type – I space-time metric. Naharaj and Martens [11] have explored a class of interior solutions with homogeneous density source and a specified form of radial pressure Coley and Tupper [5] have prove that the anisotropc fluid space times using covariantly constant vector must satisfy many strong conditions.

In this paper we have presented some static solutions of Einstein-Maxwell's field equations for anisotropic ferrowfluid system using spherically symmetric space-time under different specific conditions. Various physical and geometrical features have been ferend and discussed.

2. The Field Equations

We take static spherically symmetric metric given by

(2.1) $$ds^2 - e^{\lambda}dr^2 - r^2 d\theta^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 + e^{\nu}dt^2$$

where λ and ν are functions of r alone

The stress energy tensor for anisotropic ferrow fluid (AEF) is given by (Herrera et. al. 1984)

(2.2) $$T^{ab} = (\rho + p_R + 2m)\nu^i \nu^j - \left(P_T + m\right)g^{ij}$$

$$+\left(P_R - P_T - 2m\right)H^i H^j$$

Where symbols have their usual meanings and H^i is the unit space like magnetic vector with

(2.3) $$\nu^i \ \nu_i = 1, \ \nu^i H_i = 0, \ H^i H_i = -1,$$

(2.4) $$H_i = h_i / h, h_i = hH_i \Rightarrow h^i h_i = -h^2$$

and

(2.5) $$2m = \mu h^2$$

where μ is magnetic permeability, ν_i is 4-velocity of fluid, h^2 is the magnitude of the magnetic field vector H_i.

The magnetic field part in the charted fluid with variable magnetic permeability is subject to satisfy Maxwell's equations given by (Ray and Banergy, 14(a))

(2.6) $\left[\mu h\left(\nu^{i} H^{i}-\nu^{j} H^{i}\right)\right] ; j=0$

The attention is confined to the commoving coordinate system for which flow vector ν^i and magnetic field vector H^i have expressions.

(2.7) $\nu^{i}=\delta_{4}^{i} e^{-\nu / 2}$,

(2.8) $H^{i}=\delta_{4}^{i} e^{-\nu / 2}$,

Thus under commoving system the Maxwell equations (2.6) are solved by using (2.3) and (2.4) to evaluate the magnitude of the magnetic field h^2 and variable magnetic permeability μ as

(2.9) $h^{2}=\psi^{4} / r^{4}$

(2.9a) $\mu=\phi^{2} / \psi^{2}$

where ϕ is constant of integration and ψ is function of r. Therefore the equation (2.5) provides the value

(2.10) $m=(1 / 2) \mu h^{2}=(1 / 2)\left[\left(\phi^{2} \psi^{2} / r^{4}\right)\right]$

The assumption of static spherical symmetry under commoving coordinate system restricts the components of the stress-energy tensor (2.2). Accordingly we have

$$T_{1}^{1}=-\left(P_{R}-m\right),$$

$$T_{2}^{2}=-\left(P_{r}+m\right)=T_{3}^{3}$$

and $$T_{4}^{4}=(\rho+m)$$

Thus the Einstein field equations for the anisotropic ferrofluid system $R_{ij}-\frac{1}{2} R g_{ij}=-K T_{ij}$ described through the energy momentum tensor (2.2) generates the following equations

(2.11) $K\left(P_{R}-m\right)=e^{-\lambda}\left[\left(\nu^{\prime} / r\right)+\left(1 / r^{2}\right)\right]-1 / r^{2}$, (2.12) $K(P_r + m) = e^{-\lambda}\left[(\nu''/2) - (\lambda'\nu'/4) + \left(\nu'^2/4\right) + \left(\nu' - \lambda'/2r\right)\right],$

(2.13) $K(\rho + m) = e^{-\lambda}\left[(\lambda'/r) - (1/r^2)\right] + 1/r^2,$

Here the prime denotes derivative with respect to r

The sum of equation (2.11) and (2.13) gives

(2.14) $K(\rho + P_R) = e^{-\lambda}\left[(\lambda'/r) + (\lambda'/r)\right]e^{-\lambda}\left[\left(\nu' + \lambda'/r\right)\right]$

On subtracting the equation (2.12) from the equation (2.11) we have

(2.15) $K(P_R - P_r) = 2Km - e^{-\lambda}$

$$\left[\nu'/2) + (\nu'^2/4) - (\lambda'\nu'/4) - \lambda'/2r - \nu'/2r - 1/r^2\right] - 1/r^2$$

On differentiating the equation (2.4) with respect to r and using the equation (2.10) one can get

(2.16) $KP_R' = e^{-\lambda}\left[(\nu''/r) - (\nu'/r^2) - 2/r^3 - (\lambda'\nu'/r) - \lambda'/r^2\right] +$

$$2/r^3 + \left[(2/r) - (\psi'/\psi)\right](-2Km)$$

Further using the equation (2.15) we find

(2.17) $KP_R' = -(1/2)e^{-\lambda}\nu'(\lambda'\nu')/r - K(P_R - P_r)(2/r) + 2Km(\psi'/\psi)$

Thus the equation (2.14) and (2.16) provide

(2.18) $P_R' + (\rho + P_R)(\nu'/2) + (P_R - P_r)(2/r) - (\phi^2\psi\psi/r^4) = 0$

This equation describes the factors affecting the rate of variation in radial pressure.

3. Solution of the Field Equations

Model – I

Here we start with a physical plausibility that the matter distribution consists of an ionized imperfect gas satisfying the equation of state $P_R \propto \rho$. This situation can be well described through the setting

(3.1) $K_\rho = \alpha e^{-\lambda}/r^2$

(3.2) $KP_R = \beta e^{-\lambda} / r^2$,

where α and β positive constants. The value of ν is obtained by integration of equation (2.14) as

(3.3) $\nu = K\int(\rho + P_R)e^{\lambda}rdr - \lambda + \log k_1$,

where k_1 is a constant of integration.

The equation (3.1) and (3.2) simplify equation (3.3) to

(3.4) $\nu = \log\left(k_1 r^k e^{-\lambda}\right)$,

where $k = \alpha + \beta$

By making use of the values of m and P_R from equation (2.10) and (3.2) in equation (2.15), we get

(3.5) $r^2 e^{-\lambda}\left(\lambda'' - \lambda'^2\right) + (3/2)kre^{-\lambda}\lambda' - (1/2)(k^2 - 4i - 4)e^{-\lambda} =$

$2 - 2KP_r r^2 - 2K\left(\phi^2\psi^2 / r^2\right)$

Now to solve the field equations, we have two cases

Case I :

He we suppose that tangential pressure be expressed in the form

(3.6) $KPT = k_2 e^{-\lambda} / r^2$

with k_2 as constant.

This shows that $P_T \to 0$, as $r \to \propto$

By choosing $y = e^{-\lambda}$ the equation (3.5) reduces to

(3.7) $r^2 y'' + (3/2)kry' + (1/2)(k^2 - 4i - 4k_2 - 4)y = 2K(\phi^2\psi^2 / r^2) - 2$

By choosing new variable z as $r = e^Z$ leads the equation (3.7) to

(3.8) $d^2y / dz^2 + (1/2)(3k - 2)dy / dz + (1/2)(k^2 - 4\alpha - 4k_2 - 4)$

$y = 2K(\phi^2\psi^2)e^{-2z} - 2$

This equation provides the general solution in the form given by

(3.9) $y = Ae^{P1z} + Be^{p2z} + k_3 + k_4 e^{-2z}$

where A and B are constants of integration and P_1, P_2, k_3, k_4 are constants given by

(3.10) $P_1 = -(1/4)\left[3k - 2 - \sqrt{k^2 + 20\alpha - 12\beta + 36 + 32k_2}\right]$

(3.11) $P_2 = -(1/4)\left[3k - 2\sqrt{k^2 + 20\alpha - 12\beta + 36 + 32k_2}\right]$

(3.12) $k_3 = -4/(k^2 - 4\alpha - 4k_2 - 4)$

(3.13) $k_4 = 4K(\phi^2\psi^2)/(k^2 - 10\alpha - 6\beta - 4k_2 + 8)$

(3.14) $y = e^{-\lambda} = Ar^{p1} + Br^{p2} + k_3 + k_4/r^2$

The value of e^v is is then obtained from the equation (3.4) as

(3.15) $e^v = k_1\left[Ar^{p1-C} + Br^{p2+C} + k_3 r^C + k_4 r^{(C-2)}\right]$

Hence the space-time metric in this case reads as

(3.16) $$ds^2 = -\left[Ar^{P_1} + Br^{P_2} + k_3 + k_4/r^2\right]^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + k_1\left[Ar^{P_1+C} + Br^{p_2+C} + k_3 r^C + k_4 r^{(C-2)}\right]dt^2$$

This describes the static spherically symmetric model consistent with the ferrofluid system.

Implications of the solution (3.16)

1. The physical requirements

(3.17) $\rho \geq 0, R_R \geq 0, P_T \geq 0.$

Imply the conditions on α, β and k_2 as

(3.18) $\alpha \geq 0, \beta \geq 0, k_2 \geq 0$

2. the values of kinematical parameters associated with time like unit flow vector v^i corresponding to (3.16) are given by

(3.19) Expansion : $v_1^2 = 0$

(3.20) Shear : $\sigma^2 = 0$,

(3.21) Rotation : $\omega^2 = 0$,

(3.22) Acceleration $\equiv v^2 = -\left(\nu'^2 / 4\right)e^{-\lambda}$

where v is given by the equation (3.15) and $e^{-\lambda}$ by the equation (3.14).

3. the choice $\beta = k_2$ implies that the radial and tangential pressures are equal. The restriction $\beta = k_2 = 0$ leads the solution (3.16) to a dust filled universe. The selection $\alpha = 3/3 = 3k_2$ procures the radiating model. When one identifies $\alpha = \beta = k_2$, then it reduces to a super dust model.

4. the metric (3.16) with $\beta = -\alpha = k_2 = 0$ and $k_1 = 1$ assumes the solution in the form

(3.23) $$ds^2 = -\left[1 + Ar^2 + B/r + K\phi^2\psi^2/2r^2\right]^{-1} dr^2 -$$

$$r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + [1 + Ar^2 + B/r + K\phi^2\psi^2/2r^2]dt^2$$

This describes the static exterior field of the anisotropic ferrofluid distribution.

5. For A = 0 the solution (3.23) reduces to Reissner-Nordstrom metric describing the gravitational field in the exterior region of a infinitely conducting static sphere which has the metric form

(3.24) $$ds^2 = -\left[1 + (B/r) + K\phi^2\psi^2/2r^2\right]^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) +$$

$$\left[1 + (B/r) + K\phi^2\psi^2/2r^2\right]dt^2$$

6. A metric (3.23) with $A = 0 = \phi = \psi$ generates the well known Schwarchild's exterior solution

(3.25) $$ds^2 = -\left[1 + (B/r)\right]^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + \left[1 + (B/r)\right]dt^2$$

7. A metric (3.23) reduces to deSitter's model for a static homogeneous universe for the substitution $B = 0 = \phi = \psi$ given by

(3.26) $$ds^2 = -\left[1 + Ar^2\right]^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + \left[1 + Ar^2\right]dt^2$$

8. The selection A = B = 0 reduces to solution (3.23) to the line element characterizing gravitational field of an electron (Eddington, 8).

(3.27) $$ds^2 = -\left[1 + K\phi^2 f^2 / 2r^2\right]^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) +$$

$$\left[1 + K\phi^2 f^2 / 2r^2\right]dt^2$$

9. The treatment B = 0 in the metric (3.23) generates the line element as obtained by Aherkar and Asgekar [1] representing a static spherically symmetric space time model for the universe filled with the magnetofluid.

(3.28) $$ds^2 = \left[1 + Ar^2 + k\phi^2\psi^2 / 2r^2\right]^{-1} dr^2 - r^2$$

$$\left(d\theta^2 + \sin^2\theta d\phi^2\right) + \left[1 + Ar^2 + k\phi^2\psi^2 / 2r^2\right]dt^2$$

Case 2 :

Here we suppose that the tangential pressure has the form as

(3.29) $$P_T = r^{C_5}\left[rF_1' + r(\log r)^2(F_1' + rF_1'') + k_6 rF_1' \log r\right]$$

where F_1 is any arbitrary function of r and the constants k_5 and k_6 are

(3.30) $$k_5 = (3/4)(k-2) + (1/4)\sqrt{4\alpha + 36\beta - 5k^2 + 12},$$

(3.31) $$k_6 = (1/2)\sqrt{4\alpha + 36\beta - 5k^2 + 12}$$

Thus the equation (3.5) reduces to

(3.32) $$r^2 y'' + (3/2)kry' + (1/2)(k^2 - 4\alpha - 4)y = 2K(\phi^2\psi^2)$$

$$-2 + K_r^{C_5+2}\left[rF_3' + r(\log r)^2(F_3' + rF_1'') + k_6 rF_1' \log r\right]$$

where $y = e^{-\lambda}$

on putting $r = e^2$ this gives

(3.33) $d^2y/dz^2 + (1/2)(3k-2)dy/dz + (1/2)$

$$(k^2 - 4\alpha - 4)y = 2K(\phi^2\psi^2)e^{-2z} - 2 + 2Ke^{(C_5+2)}[\eta'' + k_6\eta']$$

where $\eta(z)$ is the value of $E_1(r)$ obtained by putting $r = e^{z.}$

This equation (3.33) admits the general solution in the form

(3.34) $y = \bar{\alpha}e^P 3^z + \bar{\beta}e^{P_{4z}} + + k_7 + k_8 e^{-2z} - 2Ke^{[C_5+2)z}\eta,$

where $\bar{\alpha}$ and $\bar{\beta}$ are constants with

(3.35) $P_3 = -(1/4)\left[(3k-2) - \sqrt{k^2 + 20\alpha - 12\beta + 36}\right],$

(3.36) $P_4 = -(1/4)\left[(3k-2) + \sqrt{k^2 + 20\alpha - 12\beta + 36}\right],$

(3.37) $k_7 = -\left[4/(k^2 - 4\alpha - 4)\right]$

and

(3.38) $k_8 = 4K(\phi^2\psi^2)/(k^2 + 10\alpha - 6\beta + 8)$

On putting $e^2 = r$ in the equation (3.34) gives

(3.39) $y \equiv e^{-\lambda} = \bar{\alpha}r^{p_3}\bar{\beta}r^{p_4} + k_7 + k_8/r^2 - 2Kr^{(C_5+2)}F_1.$

The value of e^{λ} is then obtained from the equation (3.4) as

(3.40) $e^v = k_1\left[\bar{\alpha}r^{P_3+C} + \bar{\beta}r^{p_4-C} + K_7r^C + K_8r^{C-2} - 2Kr^{(C_5-C-2)}F_1\right]$

Hence the space-time metric in this case is given by

(3.41) $ds^2 = -\left[\bar{\alpha}r^{P_3} + \bar{\beta}r^{P_4} + k_7 + k_8/r^2 - 2Kr^{(C_5+2)}F_1\right]^{-1}$

$$dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) +$$

$$k_1\left[\bar{\alpha} r^{P_3+C}+\bar{\beta} r^{P_4-C}+k_7 r^C+k_8 r^{(C-2)}-2Kr^{(C_5+C+2)}F_1\right]dt^2$$

This presents a metric structure explaining the geometrical format of the anisotropic ferrofluid system under some special choice of the function F_1.

Remarks :

For the model (3.41) the flow is essentially accelerating with the value

(3.42) $v^{*2}=-\left(\nu'^2/4\right)e^{-\lambda}$

Where *v* is given by (3.40) $e^{-\lambda}$ by the equation (3.39).

4. Case 2: (Model II)

By taking the first integral of the differential equation (2.13) the value of $e^{-\lambda}$ is written as

(4.1) $e^{-\lambda}=1-(1/r)\int\left[Kr^2\rho+K\left(\phi^2\psi^2/2r^2\right)\right]dr$

with a view to simplify this the matter density is expressed in learns of arbitrary function t(r) through a reasonable format

(4.2) $\rho=t'/r^2$

and

(4.3) $\psi^2=r^2\lambda'$

This selection with the equation (4.1) provides

(4.4) $e^{-\lambda}=T/2r^2$

where

(4.5) $T=-2Krt+2r^2-K\phi^2 r\lambda$

Further by utilizing this value of $e^{-\lambda}$ and the equation (2.10) in the differential equation (2.11) the expression for ν' is deduced as

(4.6) $\nu'=(1/Tr)\left[2r^2+Kkr^4P_R-K\phi^2\psi^2\right]-(1/r)$

With a objective to find the integral of differential equation (4.5) the value of radial pressure P_R is selected in a specific form, say

(4.7) $P_R = \in' T / 2R^3$

where $\in$ is any arbitrary function of r. Thus by introducing this value of P_R in the equation (4.5) and after integrating both sides with respect to r, the parameter ν is evaluated as

(4.8) $\nu = \int (1/Tr)\left[2r^2 - K\phi^2\psi^2\right]dr + K\in - \log r$

where the constant of integration is taken as zero. Under all the imposed plausible restrictions a class of space time models in terms of unknown functions T(r) and $\in(r)$ is given by

(4.9) $$ds^2 = -\left(2r^2/T\right)dr^2 - r^2\left(d\theta^2 + \sin\theta d\phi^2\right) +$$

$$\left(1/r\right)e^{k\in}\exp\left[\int (1/Tr)(2r^2 - K\phi^2\psi^2)dr\right]dt^2$$

Also the value of tangential pressure P_T is derived in terms of these functions, by using the equation (4.2), (4.6) and (4.7) in the equation (4.13), in the form

(4.10) $$P_r = \left(T\psi''/4r^2\right) + \left(KT\psi'^2/8r^2\right) +$$

$$\left[\left(T'/4r^2\right) + \left(Kt'/4r\right) - \left(3T/8r^3\right) + \left(1/4r\right) - \left(K\phi^2\psi\psi'/8r^3\right)\right]\in'$$

$$+t'\left[\left(1/2T\right) - \left(1/4r^2\right) - \left(K\phi^2 f^2/4r^2\right)\right] - \left(\phi^2\psi\psi'/2r^3\right) = 0.$$

By evaluating all the values of kinematical parameters with respect to the metric (4.8) it is noticed that the flow is essentially accelerating with the magnitude given by

$$v^{*2} = \left(T/8r^2\right)\left[(K/Tr)(2rt + \phi^2\lambda - \phi^2\psi^2) + K\in'\right]^2$$

or

$$v^{*2} = -\left(T/8r^2\right)\left[(K/Tr)(2rt + \phi^2\lambda - \phi^2 r^2\lambda') + K\in'\right]^2$$

Some interesting particular cases of (4.8) involving different choices of t(r) are discussed below :

Case A : The choice

(4.11) $t = (t / K) - (\phi^2 \lambda / 2)$,

Leads the radial pressure P_R to vanish.

Moreover for the choice the metric (4.8) becomes singular hence it is physically unacceptable.

Remarks :

The value $\in$ = constant also yields that the radial pressure is zero.

Case B : If the value of t is chosen in terms of unknown function ζ(r) as

$$t = (r / K) - (\phi^2 \lambda / 2) + (\zeta / \zeta')\left((\phi^2 \lambda' / 2) - (1 / K)\right),$$

or

(4.12) $t = (r / K) - (\phi^2 \lambda / 2) + (\zeta / \zeta')\left((\phi^2 \psi^2 / 2r^2) - (1 / K)\right)$,

Then the line element (4.8) gets reduced to

(4.13) $ds^2 = (\zeta / \zeta')\left[(K\phi^2 \lambda' / 2r) - (1 / r)\right]^{-1} dr^2 -$

$$r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + (\zeta / r)e^{K\epsilon}dt^2$$

Also the value of matter density ρ is given by

(4.14) $\rho = -\left(\zeta\zeta'' / 2Kr^2\zeta'^2\right)\left(K\phi^2\lambda' - 2\right) + \left(K\phi^2\lambda'' / 2Kr^2\right)\left(\zeta.\zeta'\right)$,

5. Case 3 : (Model III)

In the phase, with a view to develop a new class of models pertaining to the field equations (2.11) to (2.13) a special choice of radial and tangential pressures P_R and P_T is is made in terms of an unknown functions of $F_1(r)$ and magnetic field variables in the form.

(5.1) $P_R = (1 / r^2)(F_1 - k_{10})$

(5.2) $P_T = (1 / 2r^3)\left[F_1' r^2 - \phi^2 f f'\right]$

where k_{10} is a constant of integration

Under this selection the differential equation (2.18) gets extremely simplified as

(5.3) $(v/2)(\rho + P_R) = 0$

This gives rise to three possible sub cases as follows,

(i) $v' = 0, \rho + P_R \neq 0,$

(ii) $v' \neq 0, \rho + P_R = 0,$

(iii) $v' = 0, \rho + P_R = 0,$

Now the attentior, is focused to solve the field equations (2.11) to (2.13) with the consideration of the hypothesis (5.1) and (5.2) and these three possible cases.

Subcase (i) :

The solution under the case $v' = 0, \rho + P_R \neq 0,$

Integration of $v' = 0$ yields

(5.4) v = constant = v_0. (say)

But for small values of r, the line element should reduce to Minkowaski line element. This implies that

v = 0 at r = 0

Hence the equation (5.4) directs that $v_0 = 0$ so that the equation (5.4) gives

(5.5) v = 0

Now by utilizing equations (2.10), (5.1) and (5.5) the value of metric coefficient e^{λ} is deduced as

(5.6) $e^{\lambda} = 1 + KF_1 - Kk_{10} - \left(K\phi^2\psi^2 / 2r^2\right)$

Consequently the line element (2.1) due to the values (5.5) and (5.6) takes the form

(5.7) $ds^2 = -\left[1 + KF_1 - Kk_{10} - \left(K\phi^2\psi^2 / 2r^2\right)\right]^{-1} dr^2 -$

$r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + dt^2$

This solution represents a class of static spherically symmetric models for the universe filled with the ferrofluid. These have the following properties.

(1) It is observed from the metric (5.7) that the value

(5.8) $$r = \sqrt{\left(K\phi^2\psi^2 / 2\left(-KF_1 + Kk_{10} - 1\right)\right)}$$

forms a singularity apart from the usual singularity at r = 0. Further note that this singularity is not a coordinate singularity.

(2) For any choice of F_1 the expression for the matter density ρ is obtained by using equations (2.10) and (5.6) in the equation (2.13)

(5.9) $$\rho = -(F' / r) - \left(F_1 / r^2 + (K_{10} / r^2) - (\phi^2\psi^2 / r^4) + \left(\psi\psi' / r^3\right)\right)$$

While the expression for radial pressure, tangential pressure and magnitude of magnetic field are respectively given vide (5.1), (5.2) and (2.9) as

(5.10) $$P_R = (F_1 / r^2) - \left(k_{10} / r^2\right),$$

(5.11) $$P_T = \left(F_1' / 2r\right) - \left(\phi^2\psi\psi' / 2r^3\right),$$

(5.12) $$h^2 = (\psi^4 / r^4).$$

(3) It is clear that, for the model (5.7), the flow is expansion free, non shearing, non-accelerating and irrotational. Hence the flow lines are rigid.

6. Discussion

The choice of the functional value

(6.1) $$F_1 = k_{11}r^2 + k_{10} + \left(\phi^2 f^2 / 2r^2\right),$$

(where k_{11} is arbitrary constant)

Equates the values of radial and tangential pressures (isotropic state) and generates the class of solutions obtained by

(a)Shah [15] for magneto fluid system designed by Licherowicz [10].

(b) Aherkar and Asgekar [1] for magneto fluid system devised by Maugin [12].

For the selection

(6.2) $F_1 = k_{11} r^2 + k_{10}$

and in the absence of magnetic field the metric form (5.7) give rise to well known Einstein homogeneous static model with isotropic pressure.

7. References

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