

# NON-LINEAR PARTIAL DIFFERENTIAL EQUATION BY IMPLEMENTING STANDARD AND NON-STANDARD VARIANCE SCHEME

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**Abstract:** In this study, the nonlinear Partial Differential Equation issue is solved using our nonstandard finite variance scheme. This section of the article will introduce the lemma for the discretization equation related to the positivity and roundedness requirements. Partial Differential equations are necessary in many engineering and research fields. The physical properties of their solutions are often diverse. We systematically develop unique non-standard finite difference techniques that reproduce three of these properties. The first feature is whether the fixed points of the associated universe independent equation are stable or unstable. Numerical experiments are provided.

**Keywords:** nonlinear Partial Differential Equation, finite difference scheme, Non-standard

## 1. Introduction

Different logical orders have different ideas about explicit forms of exhibiting. Furthermore, as mathematics has always been fundamental to the physical and biological sciences, there has been a blurring of boundaries between logical orders and a resurgence of interest in both the most recent and well-established related mathematical systems. In present occasions, nonlinear Differential Equations have a ton of consideration in light of the fact that numerous physical issues in science and building are depicted scientifically by nonlinear Differential Equations in at least one than one ward/free variables. Change is the law of nature. Most things advance with time and are likewise differing and non-uniform in space. Most regular wonders are nonlinear. Linear models are those measurable models in which a progression of parameters is

masterminded as a linear blend. That is, inside the model, no parameter shows up as a multiplier, divisor or example to some other parameter. Critically, the term 'linear' in this setting does not relate to the idea of the connection between the reaction variable and the indicator variable(s), and consequently linear models are not limited to 'linear' (straight-line) connections. Linear models of such marvels are approximations of the real world and just every so often these are sensibly precise portrayals. Differential Equations that portray the way of this adjustment quantitatively by modeling each different wonders are by and large halfway and nonlinear. The shut shape arrangement of these Differential Equations regularly emerging in applications can't be acquired in spite of the fact that the presence and uniqueness of the arrangement is less demanding to set up. Thus, one is obliged to devise fitting stable numerical strategies for assurance of inexact arrangements utilizing advanced computer. Numerical strategies utilize numbers to reproduce scientific procedures, which thus as a rule recreate certifiable circumstances. These techniques have been effectively connected to think about issues in mathematics, designing, software engineering and physical sciences, for example, biophysics, air sciences and geosciences. Numerous numerical techniques have been produced to decide arrangements with a given level of exactness. Partial Differential Equations are one kind of differential equation that is used to connect the multivariable function's partial derivatives, which are unknown. There might be several independent and dependent components in this kind of function. A partial solution is an equation that can solve a certain type of Partial Differential Equations. Partial Differential equations are necessary in many engineering and research fields. The physical properties of their solutions are often diverse. We systematically develop unique non-standard finite difference techniques that reproduce three of these properties. The first feature is whether the fixed points of the associated space independent equation are stable or unstable. Non-standard single and two-stage theta approaches that are existing in the context of stiff or non-stiff differential equation organisations maintain this attribute. Schemes for the related stationary equations that adhere to the second property of energy conservation are developed by employing the non-local estimation of nonlinear responses. Combining energy-preserving structures in the space changeable with theta-methods in the time variable results in non-standard structures that, when the step sizes are appropriately connected functionally, demonstrate the solution's boundedness in addition to its positivity. Along with a spectral technique in the intergalactic variable, a suitable non-standard scheme in the period capricious is

also described. Numerical experiments are provided. We develop a nonstandard version of the finite difference structure to solve the nonlinear partial differential equation. The lemma pertaining to the positivity-to-boundedness condition for the discretization equation will be presented in this portion of the study. We simulate some numerical results in order to assess the accuracy of the lemma.

Differential Equations are utilized to express many general laws of nature and have numerous applications in physical, social, economic, and other dynamical frameworks. Specifically, the origin of the differential equation might be considered as the endeavors of Newton to represent the movement of particles. These equations may give numerous valuable data about the framework if the condition is shaped joining the different vital elements of the framework. A differential equation depicts a relation between independent, dependent variables and its derivatives. A single independent variable, a single dependent variable, and its imitative through respect to an independent flexible stand all included in ordinary differential equations. The prey-predator model, Rayleigh's equation (which has applications in fluid dynamics), the Lane–Emden equation (which has applications in astrophysics), and exponential decay or growing population model are a few well-known examples of ordinary DE. A Partial Differential Equations is equation which involves more than single independent variables as  $x_1, x_2, \dots, x_n$ ; a dependent variable  $u$  and its partial derivatives w.r.t. the independent variables such as  $F(x_1, x_2, \dots, x_n, u, \delta u / \delta x_1, \delta u / \delta x_2, \dots, \delta u / \delta x_n) = 0$ . Partial Differential Equations show up as often as possible in every aspect of engineering and physics. Moreover, in current years it is found that Partial Differential Equations have extraordinary significance in numerous areas like biology, chemistry, image processing, and graphics and in economics (finance). These Partial Differential Equations are upgraded by some extra conditions, for example, initial and boundary conditions. As the focus of our research is on Partial Differential Equations only, henceforth, we will give a brief description of Partial Differential Equations. The nonlinear A portion of In this talk we concentrate on differential partial differential equations. A novel nonstandard finite difference technique is created to construct and study a solution to the problem. Diffusion processes, which are important in a variety of disciplines, including chemistry, astronomy, metallurgy, medicine, and other areas, were studied by Evgeniya et al. (2021). The problem of charged particle dispersion in a semi-infinite thin tube under the influence of an

electromagnetic field is examined and solved in this study. Many ODE models are examined in this chapter by Wang et al. (2020) in support of epidemiological representations and the diffusion of revolution. Abdullah et al. (2013) for numerical solution of non-linear ordinary Differential Equations, a fifth order direct approach has been developed directly. Most ordinary Differential Equations research in life would simplify the problem to an ordinary Differential Equations (ODE) first-order system. They explore the traditional idea of innovation dissemination with a focus on online social networks and look at alternative ordinary differential equation models for invention diffusion. The advancement in Partial Differential Equations has been accomplished with the introduction of numerical techniques. The analysis of Partial Differential Equations is not only because of educational interest, but it has numerous applications. These Partial Differential Equations may derive from some physical problems or a model of engineering. Moreover, it is expected in most of the cases that the solution of Partial Differential Equations ought to be unique and stable under small disturbances of data. So, it is essential to have a complete analysis of the Partial Differential Equations before solving it. Aziz and Khan (2018) This study investigates a HAAR wavelet-based collocation strategy for numerical solution of diffusion and a one-dimensional and two-dimensional hyperbolic partial method for generating equations. The numerical results validate the precision, efficiency, and robustness of the proposed method. Paternoster and D'Ambrosio (2014) studied the goal of this research is to solve Partial Differential Equations that accurately and efficiently simulate a diffusion problem using computational methods for gradually tailored numerical solutions. After particular function finite differences were developed and assessed, They were used to the estimate of second order partial differentials by a realistic simulation of an equation with mixed model parameters. Gunvant (2016) uses a range of finite difference techniques to numerically solve the first initial boundary value problem (IBVP) for the semi-linear variable order fractional diffusion equation. The stability and convergence of this process are investigated using the Fourier method. Lastly, the solution to a few numerical scenarios is examined and displayed graphically using MATLAB. The objective of this research is to use progressively improved numerical solutions to solve partial differential equations that accurately and rapidly mimic the diffusion problem (D'Ambrosio and Paternoster, 2014). Morten et al. (2011) a thorough knowledge of applied mathematics, particularly differential equations and special functions is required creative

developments in sciences and technology. These are applicable to electromagnetic theory and quantum theory simulation and computing applications in, for example, photonics and nanotechnology, problem of Partial Differential Equations remains a significant subject, both at the graduate level and at the graduate level. For both temporal and spatial challenges, a numerical research shows that a general-purpose solution is significantly less precise and efficient. The nonlinear diffusion equation can be reformulated in a way that leads straight to its stochastic counterpart (Tory and Bargie, 2015). The stochastic slant aids in our understanding of the physical evolution by reproducing the schedules of molecules. Our method works very well in parallel implementation. Fallahzadeh and Shakibi (2015) used the hemitrope analysis method (HAM) to find the linear Convection Diffusion (CD) equation's series solution. Gurarslan and Sari's (2011) study offered appropriate solutions for diffusion problems that were both linear and nonlinear. Various universal quadrature techniques in space and robust stability preservation Runge-Kutta techniques were used to solve some equations. This approach can be applied to a variety of nonlinear ordinary differential equations to generate prototypes that are much more realistic.

The one-dimensional (1D) diffusion equation is a simple parabolic PDE that admits travelling wave solutions (Griffiths and Schiesser, 2012).

## 2. Objective

- To find the Non-Linear Partial Differential Equation by Implementing Standard and Non-Standard Variance Scheme
- To study the concept of Non-Linear Partial Differential Equation.

## 3. Scope of the study

An Non-Linear ordinary differential equation (ODE) is a mathematical formula that contains  $y(t)$ , its own standard derivatives, and even a specific independent variable are all unknowns. In science and mathematics, variable coefficients are used to express practically all variables, including temperature, electromagnetic potential, economic stability value, substance concentration, flow viscosity, mechanical displacement, population size of biological organisms, acoustical pressure, and some others. Numerous factors could change these figures, thus it might

be useful to ascertain how much each of them affects the unknown concentration. Physical concepts (like Newton's laws of motion) and modeling rules that explain the relationship between the unknown quantity and the components it depends on are frequently the sources of systems of equations. Consequently, we are typically given a model that represents an ODE and includes universal principles and modeling statements. We are then expected to solve the model and examine its characteristics.

#### 4. Results and Discussions

##### 4.1 Partial Differential Equation (PDE)

Many of the issues of physical science and designing fall typically into one of three actual groupings: concordance issues, eigen value issues and inducing issues. Consistently, a significant part of the time, the poor variable to any of these issues is imparted like a couple of free factors. Such issues normally, offer rising to the need of fractional subordinates in the portrayal of their direct. Numerically, a halfway differential condition (Partial Differential Equations) for a reliant variable  $(x, y, \dots)$  is a connection of structure,

$$F(x, y, \dots, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \dots, \frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y^2}) = 0$$

Where  $F$  is a given capacity of the autonomous factors  $x, y, \dots$ , the obscure capacity and of a limited number of its halfway subsidiaries.  $u(x, y, \dots)$  is an answer if after replacement of and its incomplete subsidiaries is satisfied indistinctly in a couple of regions in the space of these free factors. A Partial Differential Equations is supposed to be of solicitation  $n$  if the solicitation of the most raised halfway subordinates included is  $n$ . Condition is supposed to be straight if is direct in the dark capacity what is more, its subsidiaries and semi straight if is straight in any event the most important solicitation subordinates. Partial Differential Equations are non-straight if it is anything but direct. Straight Partial Differential Equations is supposed to be homogeneous if each term has either ward variable or one of the subsidiaries. For example, in glow equation  $u_t - ku_{xx} = 0$  is homogeneous, though, were,

$$u_t - bu_{xx} = f(x, t), \text{ here } b > 0,$$

Where  $f(x, t)$  is known function, is homogeneous equation,

##### 4.2 Non-linear partial Differential Equations

We construct the nonlinear Partial Differential calculation; a nonstandard determinate difference approach has been used. For the discretization equation, we have given the lemma relating the positivity and boundedness conditions. To ensure that the lemma is accurate, we simulate some numerical outcomes. A nonlinear partial differential equation is one that has nonlinear terms in both mathematics and physics. In mathematics, they have been applied to resolve issues like the Calabi and Poincaré conjectures. Numerous natural phenomena, including gravity and fluid dynamics, have been explained using them. They are difficult to study because, for the most part, each equation must be looked at separately and there are very few general techniques that can be applied to all of these equations. To distinguish a partial differential equation that is linear from one that is nonlinear, one usually uses the belongings of the machinist that outlines the Partial Differential Equations.

$$P. (x, y, z) Q + p (x, y, z)$$

$Q = R(x,y,z)$  And so, in nonlinear calculations such as  $z = pq$ , at least solitary stint needs to have a snowballing degree of incomplete offshoots.

A key inquiry aimed at every Partial differential equation is whether or not there is a solution and if it is unique for a given set of boundary conditions. For instance, the proving of the hardest portion of Yau's explanation to Calabi problem stood proving the existence of a Monge equation. These questions are usually fairly challenging for nonlinear equations. The Navier-Stokes equations existence (and smoothness) open problem is 1 of the 7 mathematics Millennium Star snags.

➤ **Singularities:** As with linear Partial Differential Equations, the fundamental concerns about singularities (creation, propagation, elimination, and regularity of solutions) are the same, although they are typically far more difficult to investigate. Since nonlinear Partial Differential Equations are typically not specified on arbitrary distributions, enhancements such Sobolev spaces are used in place of spaces of distributions in the linear case. The Ricci flow provides an illustration of how singularities arise. Richard S. Hamilton demonstrated that although there is short-time explanations, originalities often usage next a fixed amount of period. A thorough examination of these singularities was necessary for Grigori Perelman to

solve the Poincaré conjecture; he demonstrated how to carry the answer that transcends singularity.

- **Approximation in linear form:** Sometimes the Partial Differential Equations surrounding a known solution can be linearized in order to study the solutions in its neighborhood. Analyzing the refraction intergalactic of a fact in the moduli interplanetary of each solution is comparable to this.
- **Moduli space of the answers:** The ideal situation would be to explicitly characterize the (moduli) space of every solution, which is achievable for a very small number of extremely unique Partial Differential Equations. (In general, this is an impossible problem; for instance, it is improbable that there is a usable description of every solution to the Navier-Stokes equation because doing so would require describing every scenario in which fluid motions may occur.) The Seiberg-Witten equations are one example of such a manifold. A somewhat further intricate scenario, where the moduli interstellar stands finite-dimensional then usually explicitly compactifiable, is represented by the self-dual equations. One can even occasionally aspire to describe every solution when it comes to fully integrable models, where solutions might occasionally take the shape of a superposition of solutions.
- **Exact solutions:** While it stays rarely thinkable to characterize every solution in this way, it is frequently able to explicitly write in footings of straightforward functions. Getting down to just equations of lower dimension preferably Finding explicit solutions to ordinary Differential Equations which are often fully solvable is one way to do this.. Sometimes, this can be accomplished by searching for very symmetric solutions or by separating the variables.
- **Numerical solutions:** Almost exclusively, computer numerical solutions are utilized to obtain information about any given system of Partial Differential Equations. There is still more work to be done in numerically solving some systems, including the Navier-Stokes and other weather prediction-related equations, despite the large amount of work that has already been done.

### 4.3 An Nonstandard Finite Difference Scheme and a Nonlinear PDE

A nonlinear cubic source component in the resulting reaction-diffusion equation is analysed.



$$u_t = u_{xx} - (u - a_1)(u - a_2)(u - a_3) \quad (1)$$

Where  $a_1 = -1, a_2 = 0, a_3 = 1$  Taking these particular parameter values into consideration, the equation can be written as follows:

$$u_t = u_{xx} - u^3 + u \quad (2)$$

Before we start creating the non-standard numerical scheme, let's quickly review the key mathematical aspects of equation (2). Before proceeding to the following stage of the procedure, this will be completed. This is being done to ensure that these characteristics are present in the non-standard finite difference scheme that resolve be developed; otherwise, numerical instabilities will occur. Main, it's vital to understand that equ. (2) has three constant explanations, or stable points.

$$\bar{u}^{(1)} = -1 \quad \bar{u}^{(2)} = 0 \quad \bar{u}^{(3)} = 1 \quad (3)$$

The relationship between the 1<sup>st</sup> and 3<sup>rd</sup> fixed points is linearly stable, while the 2<sup>nd</sup> fixed point is not. The boundedness criterion is satisfied by the discretization solution to the discrete equation, as we verify by using these stable fixed-points.

$$-1 \leq u_m^n \leq 1 \Rightarrow -1 \leq u_{m+1}^n \leq 1. \quad t > 0, \quad \text{fixednallm.}$$

#### 4.4 The Standard Finite Difference Method for the Reaction-Diffusion Equation

This represents the standard finite difference approach for equation (2). For our discrete model, we employed a central variance strategy for the 2<sup>nd</sup> derivative and a onward difference scheme for the principal.

$$\frac{u_m^{n+1} - u_m^n}{\Delta t} = \frac{u_{m+1}^n - 2u_m^n + u_{m-1}^n}{(\Delta x)^2} - (u_m^n)^3 + (u_m^n) \quad (4)$$

$$u_m^{n+1} = u_m^n + \frac{\Delta t}{(\Delta x)^2} (u_{m+1}^n - 2u_m^n + u_{m-1}^n) - \Delta t (u_m^n)^3 + \Delta t (u_m^n) \quad (5)$$

We shall simulate the reaction-diffusion equation solution by the traditional finite difference method. We'll apply the following equation (5).

#### 4.5 Unusual Finite Difference Approach for Reaction-Diffusion Formula

The separate model for equation was selected as a result of earlier studies on non-standard finite difference outlines and the use of a positivity circumstance (2).

$$\frac{u_m^{n+1} - u_m^n}{\Delta t} = \frac{u_{m+1}^n - 2u_m^n + u_{m-1}^n}{(\Delta x)^2} - \left( \frac{3u_m^{n+1} - u_{m-1}^n}{2} \right) (u_{m-1}^n)^2 + u_{m-1}^n \quad (6)$$

We are competent to inscribe the denominator functions in a extra sophisticated way than in our earlier work.

$$\frac{u_m^{n+1} - u_m^n}{\frac{1-e^{-2\Delta t}}{2}} = \frac{u_{m+1}^n - 2u_m^n + u_{m-1}^n}{4\sin^2\left(\frac{\Delta x}{2}\right)} - \left( \frac{3u_m^{n+1} - u_{m-1}^n}{2} \right) (u_{m-1}^n)^2 + u_{m-1}^n \quad (7)$$

$$u^3 \rightarrow \left( \frac{3u_m^{n+1} - u_{m-1}^n}{2} \right) (u_{m-1}^n)^2 \quad (8)$$

$$u \rightarrow u_{m-1}^n \quad (9)$$

A detailed examination of equation (6) demonstrates that it behaves linearly in  $u_{m-1}^n$ . This results in the expression that follows:

$$\left[ 1 + \frac{3\Delta t}{2} (u_{m-1}^n)^2 \right] u_m^{n+1} = (1 - 2\beta)u_m^n + \beta u_{m+1}^n + \left[ \beta + \Delta t + \frac{\Delta t}{2} (u_{m-1}^n)^2 \right] (u_{m-1}^n)$$

in which  $\beta$  is described as

$$\beta = \frac{\Delta t}{(\Delta x)^2} \quad (10)$$

Several algebraic steps later, we obtain the explicit discrete equation.

$$u_m^{n+1} = \frac{(1 - 2\beta)u_m^n + \beta u_{m+1}^n + \left[ \beta + \Delta t + \frac{\Delta t}{2} (u_{m-1}^n)^2 \right] (u_{m-1}^n)}{\left[ 1 + \frac{3\Delta t}{2} (u_{m-1}^n)^2 \right]} \quad (11)$$

This discrete equation will be analysed in the section that follows.

#### 4.6 An Evaluation of Nonstandard Finite Difference Approximation

Equ. (11) must meet the boundedness criterion and the positivity condition in accordance through the nonstandard criteria aimed at finite differences. The separate variant of the positivity circumstance is called

$$0 \leq u_m^n \Rightarrow 0 \leq u_m^{n+1} \text{ fixed n all m, and} \quad (12)$$

One way to sum up the boundedness constraint is to

$$-1 \leq u_m^n \leq 1 \Rightarrow -1 \leq u_m^{n+1} \leq 1 \quad (13)$$

The condition has been met if the positivity requirement of equation (12) is satisfied.

$$1 - 2\beta \geq 0 \Rightarrow \frac{\Delta t}{(\Delta t)^2} \leq \frac{1}{2} \quad (14)$$

This is evident from the discretization equation, which is provided in equation (11), since all of the discrete solution coefficients aside from the first one in the numerator are positive. Next, we resolve demonstrate that boundedness condition of the discretization equation (11) is fulfilled, albeit with certain restrictions. For  $\beta \leq 1/2$ , equ. (11) can be inscribed as shadows:

$$u_m^{n+1} \leq \frac{\frac{1}{2}u_{m+1}^n + \left[\frac{1}{2} + \Delta t + \frac{\Delta t}{2}(u_{m-1}^n)^2\right](u_{m-1}^n)}{\left[1 + \frac{3\Delta t}{2}(u_{m-1}^n)^2\right]} \quad (15)$$

As a result, the first assumption is that, for any m and n-fixed  $-1 \leq u_m^n \leq 0$ . It follows that

$$\frac{1}{2}u_{m+1}^n \leq \frac{1}{2} \quad (16)$$

$$\frac{1}{2}u_{m-1}^n \leq \frac{1}{2} \quad (17)$$

$$\Delta t(u_{m-1}^n) \leq \Delta t(u_{m-1}^n)^2 \quad (18)$$

$$\frac{\Delta t}{2}(u_{m-1}^n)^3 \leq \frac{\Delta t}{2}(u_{m-1}^n)^2 \quad (19)$$

After integrating equations (16) and (19), we get the subsequent equation.

$$\frac{1}{2}u_m^{n+1} + \frac{1}{2}u_{m-1}^n + \Delta t \left( u_{m-1}^n + \frac{\Delta t}{2}(u_{m-1}^n)^3 \right) \leq 1 + \frac{3\Delta t}{2}(u_{m-1}^n)^2 \quad (20)$$

By separating the appearance on the right side by that expression, one can obtain equ.(20).

$$\frac{\frac{1}{2}u_m^{n+1} + \frac{1}{2}u_{m-1}^n + \Delta t \left( u_{m-1}^n + \frac{\Delta t}{2}(u_{m-1}^n)^3 \right)}{1 + \frac{3\Delta t}{2}(u_{m-1}^n)^2} \leq 1 \quad (21)$$

But the sum of the left half of the equation is just (20)  $u_{m-1}^n$ . Consequently, the supposition drawn since equ.(13) is established to stand accurate if  $\beta \leq 1/2$ . After that, we'll show that  $0 \leq u_m^n \leq 1$ , outlining  $\Omega$  as shadows:

$$u_m^n = u_{m-1}^n = u_{m+1}^n = \phi$$

Equ. (11) can be utilised to generate the following inequality:

$$u_m^{n+1} \leq \frac{\Phi - 2\beta\Phi + \beta\Phi + \beta\Phi + \Delta t\Phi + \frac{\Delta t}{2}\Phi^3}{1 + \frac{3\Delta t}{2}\Phi^2} \quad (22)$$

Given that  $u_m^{n+1} \leq 1$  is a sufficient condition for boundedness, the following inequality is probably true:

$$\frac{\Phi + \Delta t\Phi + \frac{\Delta t}{2}\Phi^3}{1 + \frac{3\Delta t}{2}\Phi^2} \leq 1 \quad (23)$$

We may infer the temporal step-size constraint thanks to this inequality, as shown below:

$$\Delta t \leq \frac{1 - \Phi}{\Phi + \frac{\Phi^3}{2} - \frac{3}{2}\Phi^2} \quad (24)$$

These inequality indications to the subsequent formulation of the period step-size limit, We further developed the nonstandard determinate difference approach for the nonlinear Partial Differential PDE. Because the new approach is basically explicit, it is straightforward to apply in order to obtain numerical solutions and it readily meets the positivity and boundedness constraints. This new scheme also includes the necessary fixed points. Numerous computer evaluations have shown that the resulting nonstandard scheme is more capable of approaching the precise solution than the traditional finite difference discretization of the issue.

## 5. Recommendations

- The standard approach to tackle fractional differential conditions is to communicate the solution as a straight mix of purported premise functions. Lots of analytical as well as numerical techniques are already recommended obtaining ways for Non-Linear Partial Differential Equations with fractional derivatives.
- Although a few limit control strategies for Partial Differential Equations-models have been explored, some others are yet neglected, for example,  $H_\infty$ -ideal limit control and Hamiltonian limit control. Since these control strategies may give new understanding in the control of assembling Partial Differential Equations-models and may concoct better regulators, it is recommended that they are examined also in future examination. This approach can tackle functions with discontinuities as well as impulse functions effectively.

## 6. Limitations of the Study

While direct methods make the actual option after a limited number of actions, iterative methods yield the actual option as a limit of sequence of approximate solutions. Beginning with an original approximation to the real option and then obtaining much better and better approximations from a computational cycle repeated time and again, for attaining a desired accuracy. As a result, in iterative technique, the quantity of computation depends upon the amount of accuracy required.

## 7. Future Scope of the study

For large system of equations, iterative methods produce solutions more quickly than the immediate techniques as the programming and information handling is much simpler for iterative

methods than for the direct methods. Nevertheless, the one drawback of Iterative solutions is the issue of choosing a good initial vector to have the iterative process, whereas no initial vector is actually needed with direct methods.

## 8. Conclusion:

This is the most sought-after feature of the calculating approach in many applications, when it comes to the mathematical explanation of the linked starting boundary value problem. The systematic literature devoted to the creation, evaluation, and use of rigorous approaches for the numerical representation of time-dependent Partial Differential Equations via nonlocal boundary conditions has garnered a lot of attention lately. This topic has been on people's minds a lot lately. Using this tried-and-true strategy, several writers have solved this classic and a few other fairly related problems within the past ten years. An unconventional finite difference technique on behalf of the Fisher equation serves as the inspiration for this paper. Below a specific functional relationship amid the period and space step sizes, the aforementioned arrangement is stable with regard to the boundedness besides positivity nose. We have suggested a methodical process for creating novel qualitatively stable systems for broad Partial Differential equations that incorporate subjective answer terms consuming a considerable meeker functional relative. In the process, we developed more universal uncomplicated stable non-standard structures of theta sort for stiff and non-stiff organisations of ODE, as well as energy-preserving outline son behalf of a class of Hamiltonian arrangements. We have created a different approach for the general diffusion equation. The stiffness chin of the linearized arrangement of Fourier factors is precisely fused into both the spectral approach (in the interstellar variable) besides the non-standard FDE (in the period variable). Our future strategy consists of two parts. First, how does the analysis in this study apply to the creation of schemes that exhibit the roundedness and positive properties of solutions to the Burger equation and the convective equation that were both taken into consideration in. The notional parts of the ethereal non-standard approach shown here will be discussed next. Numerical inclusion approaches are the only methods that can provide exact numerical reasons for the initial differential equations, tailored to each set of categorization characteristics. This is thus that the only methods available for quantitatively solving equation systems are numerical integration techniques. One of the biggest obstacles to using numerical techniques has to write a new calculation for all unique set of start or boundary values. Thus,

understanding the over-all explanation to the differential equations necessitates a lot of determination and determination. Targeted at the vast common of evils that have been studied in the domains of science and technology, numerical techniques are the only workable option. A model with discrete or infinite independent variables and dependent variables replaces a set of differential equations where both the independent variables and the dependent variables are incessant.

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