Analyzing the Distribution of Pre-test Estimator of Error Variance in Linear Regression Model

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Abstract

In statistical inference, the estimation of error variance plays a crucial role in regression analysis and hypothesis testing. Using the available extraneous information, some improved estimators of error variance in the linear regression model may be constructed which combine both the sample and the non-sample information. A pre-test estimator of error variance is considered when prior information about regression parameters is available, influencing the estimation process. The modest aim of this article is to study and analyze distributional properties of a pre-test estimator of error variance in a linear regression model with numerical computation.

Keywords: Ordinary Least Squares Estimator (OLS), Restricted Least Squares Estimator (RLS), Pre-Test Estimator.

AMS Mathematics Subject Classification (2010) - 62J05

Introduction

Statistical inference in linear regression model heavily depends upon, among other things, the properties of error variance. For instance, while carrying out statistical tests of significance or constructing confidence intervals for the parameters the knowledge of error variance is inevident. In situation where error variance is not known, it has to be estimated from the available sample information and is constructed using residual sum of squares. Generally, the error variance are estimated using least squares estimates of the coefficients in the model. However, following Stein's (1964) elegant proof of the inadmissibility of the usual least square estimator of variance, extensive work has been reported on improved estimation of error variance. In a very interesting article Matta and Casella (1990) reviewed and examined the developments in variance estimation under decision theoretic set up. Ohtani (1987, 2001) working with the iterative Stein rule estimator of error variance demonstrated that under squared error loss it is dominated by the least squares based variance estimator when number of regressors is atleast five. Dube et. Al (2015) studied the performance properties of disturbance variance under restrictions using LINEX loss function. Clarke et. al (1987a, b) using coverage probabilities, worked on Pre-test estimators of error variance. The sampling performance and exact probability distribution of pre-test estimators under inequality estimators have been explored by Wan (1997). In the field of Pre-test estimator in two variable model, Khan and Saleh (1997), Khan et al. (2002) and Khan et. al (2005) studied the performance properties of the pre-test estimator of the intercept parameter of simple linear regression model. Their study shows that under certain conditions pre-test estimator dominates the least squares estimator. Kumar (2016) derived the exact distribution of Pre-test estimator of regression coefficient under orthonormal regression model.

The aim of this paper is to find the distribution of Pre-test estimator of disturbance variance in linear regression model. The organization of this paper is as follows: Section 2 describes the model and estimators, testing of hypothesis and Pre-test estimator of disturbance variance. In Section 3 the distribution function of the Pre-test estimator of error variance has been determined then using distribution function, the density function of Pre-test estimator of error variance has been obtained. Numerical computation and comparison are made in section 4. Lastly, a brief outline of proof of the theorem is provided in Appendix.

2 The Model and The Estimators

Consider the multiple linear regression model

$$y = X\beta + \varepsilon \tag{2.1}$$

where y is an $n \times 1$ vector of observations on the response variable, X is an $n \times p$ full column rank nonstochastic matrix of n observations on p explanatory variables, β is a $p \times 1$ vector of unknown parameters associated with the p regressors and ε is an $n \times 1$ vector disturbances. The elements of the disturbances vector ε are assumed to be independently and identically distributed each following normal distribution with mean zero and variance σ^2, so that $E(\varepsilon) = 0$ and $E(\varepsilon\varepsilon') = \sigma^2 I_n$ where σ^2 is finite but unknown. Suppose some prior information in the form of restrictions on β are available and is given by

$$r - R\beta = \delta \tag{2.2}$$

where R is $q \times p$ $(q \leq p)$ matrix of known elements with rank q, q being the number of restrictions imposed on the coefficients, r is a $q \times 1$ vector of known elements and δ is $q \times 1$ vector representing the errors in the restrictions. The least squares estimator without and with restrictions are given by

$$b = (X'X)^{-1}X'y \qquad \text{and} \tag{2.3}$$

$$b_R = (X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}(r - Rb) \tag{2.4}$$

As σ^2 is generally unknown, it has to be estimated from the available sample information. In view of this, the following estimators of error variance, one entirely based on the sample information and the other utilizing prior information may be constructed. Thus using the ordinary least squares estimator (2.3) and restricted least squares estimator (2.4) the following estimators of error variance are constructed

$$s^2 = \frac{1}{m}e'e\,; \qquad m = n - p + \theta \tag{2.5}$$

$$s_R^2 = \frac{1}{m'}\, e_R' e_R\,; \qquad m' = n - p + q + \theta \tag{2.6}$$

Here e and e_R are the residual vector obtained using ordinary least squares and restricted least squares estimator respectively, and *m and m'* are the arbitrary scalars characterizing the estimators. It may be noted that the residual vector e_R using (2.4) can be written as $e_R = y - X\, b_R$ which further using (2.1) and (2.4) reduces to

$e_R = \bar{M}_X\, \varepsilon - X(X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}\delta$ where

$\bar{M}_X = (I - X\,\Omega^{-1}X')$; $\Omega^{-1} = (X'X)^{-1} - (X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}R(X'X)^{-1}$

It is easy to verify that $\bar{M}_X$ is an idempotent matrix with rank $v + q$.

In order to test the compatibility of the sample information (2.1) and the non-sample information (2.2), the following test of hypothesis may be performed.

$$H_0 :\ \delta = 0 \tag{2.7}$$

against

$$H_1 :\ \delta \neq 0 \qquad \text{where } \delta = R\beta - r. \tag{2.8}$$

This hypothesis is tested using the Wald test statistics, given by

$$u = \frac{(r-Rb)'\left[R\left(X'X\right)^{-1}R'\right]^{-1}(r-Rb)/q}{e'e/v} \tag{2.9}$$

The test statistic u has a central F distribution with q and v degrees of freedom and under (2.7) , the test statistics u has a non-central F distribution with q and v degrees of freedom with non-centrality parameter λ. As λ is typically unknown, it is usual to test under the null hypothesis H_0, and so reject the hypothesis if $u > F^{\alpha}_{(q,v)} = c$, where c is the critical value and is determined for a given significance level of the test

$$\int_c^{\infty} dF(u) = \alpha$$

In view of this, a Pre-test estimator of error variance may be defined which is given by

$$s^2_{PT} = I_{[0,c)}(u)\, s^2_R + I_{(c,\infty]}(u)s^2 \tag{2.10}$$

where c is the critical value of the Pre-test and $I(.)$ is an indicator function which is one if u falls in the given interval and zero otherwise.

3 Distribution of Pre-Test Estimator of Error Variance

In order to determine the distribution function of pre-test estimator s^2_{PT}, the statistics u in (2.9) can be written as

$$u = \frac{u_1/q}{u_2/v}\ ; \qquad v = n - p \tag{3.1}$$

where $$u_1 = \frac{(r-Rb)'\left[R(X'X)^{-1}R'\right]^{-1}(r-Rb)}{\sigma^2} \tag{3.2}$$

and $$u_2 = \frac{e'e}{\sigma^2} \tag{3.3}$$

It is clear that under H_0, u_1 follows a central chi-square distribution with q degrees of freedom while under H_1 it follows a non-central chi-square distribution with non-centrality parameter $\frac{\lambda}{2}$. It is also well recognized that u_2 is distributed independently of u_1 and follows a central chi-square distribution with v degrees of freedom[see Searle (1971)]. First write s_R^2 and s^2 in the manner following Ohtani (2002):

$$s_R^2 = \frac{1}{m'}\{e'e + (r-Rb)'[R(X'X)^{-1}R']^{-1}(r-Rb)\} = \frac{\sigma^2}{m'}(u_1 + u_2)$$

while $$s^2 = \frac{\sigma^2}{m}\, u_2$$

Thus, using these, the Pre-test estimator (2.10) becomes

$$s_{PT}^2 = I\left[\frac{u_1}{u_2} < c^*\right]\frac{\sigma^2}{m'}(u_1 + u_2) + I\left[\frac{u_1}{u_2} > c^*\right]\frac{\sigma^2}{m}\, u_2 \tag{3.4}$$

where $c^* = \frac{qc}{v}$, $m = v + \theta$ and $m' = v + q + \theta$

To determine probability density function of Pre-test estimator s_{PT}^2, first determine the distribution function. The following theorem gives the distribution function of the Pre-test estimator (2.10)

Theorem 1: When errors in the model (2.1) are normally distributed, the distribution function of the Pre-test estimator s_{PT}^2 is given by

$$F(\tau) = \sum_{i=0}^{\infty} w_i(\lambda)\, G\left(\frac{v+q}{2} + i,\ \frac{m'\tau}{2\,\sigma^2}\right) \mathrm{I}_{\frac{qc}{v+qc}}\left(\frac{q}{2} + i,\ \frac{v}{2}\right)$$
$$+ \sum_{i=0}^{\infty} w_i(\lambda)\frac{1}{B\left(\frac{q}{2}+i, \frac{v}{2}\right)} \int_{\frac{qc}{v+qc}}^{1} g^{\frac{q}{2}+i-1}(1-g)^{\frac{v}{2}-1}\; G\left(\frac{v+q}{2} + i,\ \frac{m\,\tau}{2\,\sigma^2(1-g)}\right) dg \tag{3.5}$$

where $w_i(\lambda) = \frac{e^{-\lambda}\lambda^i}{i!}$, $G(a, x)$ is the incomplete gamma function and $\mathrm{I}_x(a, b)$ is the incomplete Beta function.

Proof: See Appendix.

From the above theorem, the density function of the Pre-test estimator may be obtained by differentiating $F(\tau)$ with respect to τ. For this purpose the following formula is used owing to the involvement of τ in the incomplete gamma function:

$\frac{\partial}{\partial x} G(a,x) = \frac{1}{\Gamma a} x^{a-1} e^{-x}$ [see; Abramowitz and Stegun (1972)].

Hence, the probability density function of the Pre-test estimator s_{PT}^2 is given by

$$f(\tau) = \sum_{i=0}^{\infty} w_i(\lambda) \left(\frac{m'}{2\,\sigma^2}\right)^{\frac{v+q}{2}+i} \frac{1}{\Gamma\left(\frac{v+q}{2}+i\right)} . \mathrm{I}_{\frac{qc}{v+qc}}\left(\frac{q}{2}+i, \quad \frac{v}{2}\right)$$

$$\times \tau^{\frac{v+q}{2}+i-1} e^{-\left(\frac{m'}{2\,\sigma^2}\right)\tau} + \sum_{i=0}^{\infty} w_i(\lambda) \frac{1}{\Gamma\left(\frac{q}{2}+i\right)\Gamma\left(\frac{v}{2}\right)} \left(\frac{m}{2\,\sigma^2}\right)^{\frac{v+q}{2}+i} \tau^{\frac{v+q}{2}+i-1}$$

$$\times \int_{\frac{qc}{v+qc}}^{1} \frac{g^{\frac{q}{2}+i-1}}{(1-g)^{\frac{q}{2}+i+1}} \quad e^{-\left(\frac{m}{2\,\sigma^2(1-g)}\right)\tau} dg \qquad (3.6)$$

The equation (3.6) is quite intricate and no clear conclusion about the shape of the distribution can be made. Hence, the probability density function of the statistic s_{PT}^2 has been computed empirically using MATLAB. For this, firstly the variable g is transformed using $g = \frac{h}{1+h}$ to a new variable and again making change of variable $\left(\frac{m\,\tau}{2\,\sigma^2}\right) h = h_1$ and using the Incomplete Gamma function reducing (3.6) to

$$f(\tau) = \sum_{i=0}^{\infty} w_i(\lambda) \left(\frac{m'}{2\,\sigma^2}\right)^{\frac{v+q}{2}+i} \frac{1}{\Gamma\left(\frac{v+q}{2}+i\right)} . \mathrm{I}_{\frac{qc}{v+qc}}\left(\frac{q}{2}+i, \ \frac{v}{2}\right) \times \ \tau^{\frac{v+q}{2}+i-1} e^{-\left(\frac{m'}{2\,\sigma^2}\right)\tau}$$

$$+ \sum_{i=0}^{\infty} w_i(\lambda) \frac{1}{\Gamma\left(\frac{v}{2}\right)} \left(\frac{m}{2\,\sigma^2}\right)^{\frac{v}{2}} \left[1 - G\left(\frac{q}{2}+i, \ \frac{m'\tau}{2\,\sigma^2} . \frac{qc}{v}\right)\right] \tau^{\frac{v}{2}-1} e^{-\left(\frac{m}{2\,\sigma^2}\right)\tau} \qquad (3.7)$$

4. Numerical Computation and comparison

Now it is feasible to compute the equation (3.7) easily and using it for determine the shape of the probability density function. Owing to intricacy of the expression (3.7), it is computed for some specific values of the parameters. For this purpose few selected values of s_{PT}^2 (or τ) and λ for fixed degrees of freedom and the critical value c. The values of number of restrictions are also varied and taken to be $q = 1, 2 \ and \ 5$. These values are tabulated in Table 1. It may be noted from the Table 1 corresponding to the values λ=0, provides the probability distribution of s_R^2. Clearly, as moving down in the Table, i.e., increase the values of restrictions or prior information the probability of s_{PT}^2 moving away from the value decreases indicating that the availability of prior information enhances the chances of the value to be around true parametric value. However as moving horizontally in the Table, i.e., increase the value of the non-centrality parameter (increase the uncertainty), the probabilities decrease indicating that the chances of s_{PT}^2 to be closer to the true value of the parameter decrease. This seems justified in the sense that incorporation of a correct prior information is always beneficial.

Table 1 Density function of Pre-test estimator for selected values of q, λ and fixed $v = 10$

v=10										
			1	5		5				)
q=1		0	0	0	0	0	0	0	00	00
		5	5	5	5	5	4	3	01	00
		8	2	8	8	4	3	8	27	36
		8	9	2	6	4	0	5	16	72
		6	2	9	3	7	5	8	75	33
		3	3	0	7	9	5	1	40	08
		8	8	8	8	7	6	4	04	01
		3	3	3	3	3	3	2	01	00
		1	1	1	0	0	0	0	00	00
q=2		0	0	0	0	0	0	0	00	00
		2	1	0	9	4	8	6	13	02
		3	5	4	6	5	2	9	04	74
		4	5	8	3	0	3	4	08	25
		6	3	4	3	0	8	2	26	55
		4	4	2	9	2	9	7	70	19
		2	1	1	1	1	0	8	08	03
		4	4	4	4	4	3	3	02	01
		0	0	0	0	0	0	0	00	00
q=5		0	0	0	0	0	0	0	00	00
		0	9	9	8	6	3	8	07	01
		1	5	3	5	3	4	3	55	03
		0	2	1	2	9	8	5	83	69
		9	8	2	4	0	1	6	65	68
		4	4	3	1	7	0	7	52	65
		4	4	4	4	4	4	3	17	09
		4	4	4	4	4	4	4	03	02
		0	0	0	0	0	0	0	00	00

In order to get a clearer picture these values are plotted firstly for a given λ and then the values of λ are varied for a fixed value of q.

Figure 1(A) Probability Distribution of the Pre-test estimator of Error Variance for $\sigma^2 = 1,\ v = 10$, and for fixed λ

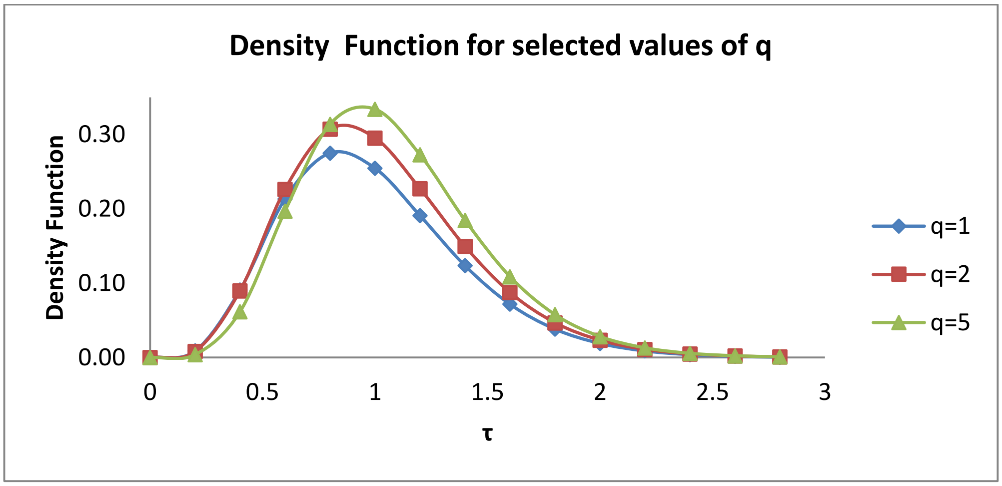
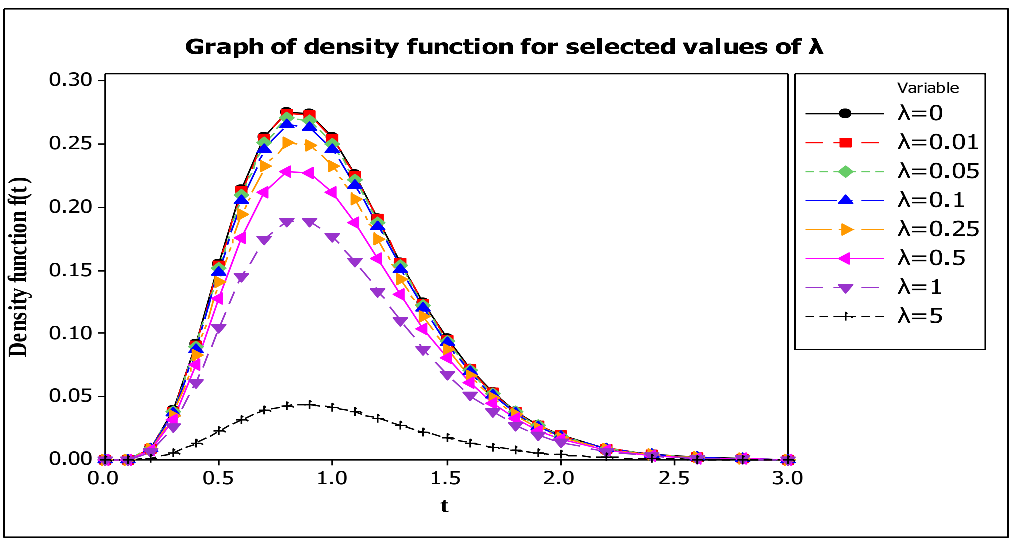


Figure 1(B) Probability Distribution of the Pre-test estimator of Error Variance for $\sigma^2 = 1,\ v = 10, q = 1$



It is clear from the Figure 1(A), that the distribution is unimodal and skewed and as the number of restrictions or prior information increase the probability distribution shifts to the left with modal value also shifting toward the assumed value $\sigma^2 = 1$. Next, the distribution of s_{PT}^2 has been plotted for varying values of λ (see Figure 1(B)). It is interesting to observe that from this figure that when the value of non-centrality parameter increases, the kurtosis of the distribution increases while there is not much affecting the skewness of the distribution i.e. it changes from being Leptokurtic to Platykurtic. In fact, it is observed that as the value of λ is very large, the distribution become flatter, i.e., the ability of pre-test estimator to differentiate between alternative values of σ^2 decline. The Figures 1 (B_1) and 1(B_2) indicate that for very small values of λ (0 < λ< 0.1) the probability density function are almost coincident while a significant difference is obtained when the values of λ are large.

Figure 1 (B₁) Probability Distribution of the Pre-test estimator of Error Variance for σ^2= 1, $v = 10, q = 1$

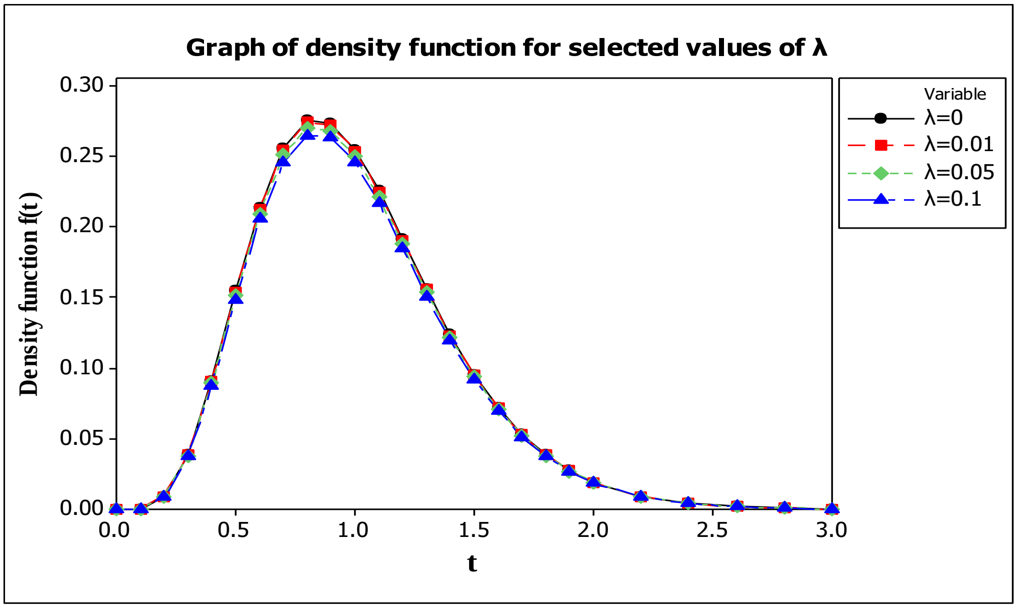
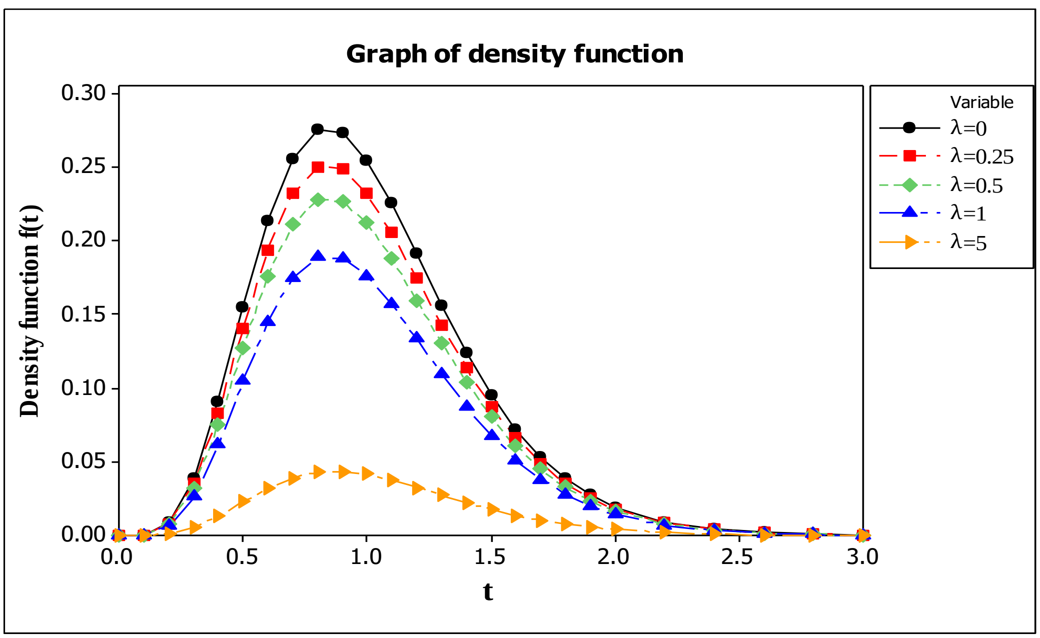


Figure 1 (B₂) Probability Distribution of the Pre-test estimator of Error Variance for σ^2= 1, $v = 10, q = 1$



Next, the effect of the variation in degrees of freedom is carried out. In the Table 2, the probability distribution of s_{PT}^2 has been enumerated for various values of v. The Figure 2 is devoted to portray the behavior of the distribution of s_{PT}^2 for various values of v.

An interesting result may be deduced from the Table 2 and also from the Figure 2 that as the degrees of freedom increase, the distribution becomes more peaked and the probability of s_{PT}^2 to hover around the specified value of the parameter increase, indicating that larger the observations, more are the chances to get towards the true value of the parameter.

Table 2 Probability Distribution of the Pre-test estimator of Error Variance for selected values of v and fixed q, λ

τ	v=10	v=20	v=30	v=50	v=100
0.0	0.00000	0.00000	0.00000	0.00000	0.00000
0.1	0.00034	0.00000	0.00000	0.00000	0.00000
0.5	0.10483	0.05932	0.02816	0.00535	0.00006
1.0	0.17648	0.26232	0.32703	0.42843	0.61276
1.5	0.06782	0.06284	0.04883	0.02486	0.00335
2.0	0.01408	0.00451	0.00121	0.00007	0.00000
2.2	0.00677	0.00128	0.00020	0.00000	0.00000
2.4	0.00311	0.00034	0.00003	0.00000	0.00000
2.6	0.00138	0.00008	0.00000	0.00000	0.00000
2.8	0.00059	0.00002	0.00000	0.00000	0.00000
3.0	0.00025	0.00000	0.00000	0.00000	0.00000

Figure 2 Probability Distribution of the Pre-test estimator of Error Variance for $\sigma^2 = 1$, $q = 1$ and for fixed λ

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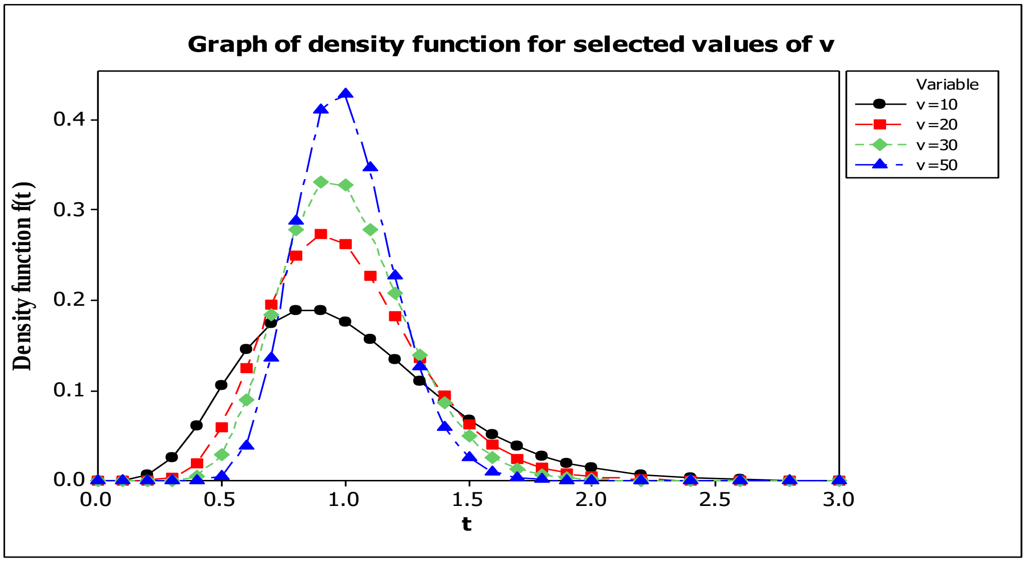

Appendix

Proof of Theorem 1:

The distribution function of the pre-test estimator is given by

$$F(\tau) = P[s_{PT}^2 \le \tau] = P\left[\frac{\sigma^2}{m'}(u_1+u_2) \le \frac{\tau}{\frac{u_1}{u_2}} \le c^*\right] P\left[\frac{u_1}{u_2} \le c^*\right]$$

$$+P\left[\frac{\sigma^2}{m}(u_2) \le \tau / \frac{u_1}{u_2} > c^*\right] P\left[\frac{u_1}{u_2} > c^*\right] \quad \text{(A.1)}$$

since $u_1 \sim \chi^2_{(q,\ \lambda)}$ *and* $u_2 \sim \chi^2_v$, so their density functions are given by

$$f(u_1) = \sum_{i=0}^{\infty} w_i(\lambda) \frac{u_1^{\frac{q}{2}+i-1}}{2^{\frac{q}{2}+i}\,\Gamma(\frac{q}{2}+i)} e^{\frac{-u_1}{2}} \text{ and } f(u_2) = \frac{u_2^{\frac{v}{2}-1}}{2^{\frac{v}{2}}\,\Gamma(\frac{v}{2})} e^{\frac{-u_2}{2}}$$

Using these above density functions, equation (A.1) may be written as

$$F(\tau) = \iint_{R_1} f(u_1).f(u_2)du_1\,du_2 + \iint_{R_2} f(u_1).f(u_2)du_1\,du_2 \quad \text{(A.2)}$$

where R_1 and R_2 are the regions given by

$\left\{(u_1,u_2): u_1+u_2 \le \frac{m'\tau}{\sigma^2},\ \frac{u_1}{u_2} \le c^*\right\}$ and $\left\{(u_1,u_2): u_2 \le \frac{m\,\tau}{\sigma^2},\ \frac{u_1}{u_2} > c^*\right\}$ respectively.

Now let us solve the first term of (A.2) i.e

$$\iint_{R_1} f(u_1).f(u_2)du_1\,du_2 = \sum_{i=0}^{\infty} w_i(\lambda) \frac{1}{2^{\frac{v+q}{2}+i}\,\Gamma\left(\frac{q}{2}+i\right)\Gamma\left(\frac{v}{2}\right)} \iint_{R_1} u_1^{\frac{q}{2}+i-1} u_2^{\frac{v}{2}-1} e^{-\left(\frac{u_1+u_2}{2}\right)} du_1\,du_2$$

To solve it, making the change of variables $u_1 + u_2 = t_1$ and $\frac{u_1}{u_2} = t_2$ so the value of Jacobean is given by $|\, J\,| = \frac{t_1}{(1+t_2)^2}$, therefore it reduced to

$$\iint_{R_1} f(u_1).f(u_2)du_1\, du_2 = \sum_{i=0}^{\infty} w_i(\lambda)\ \frac{1}{2^{\frac{v+q}{2}+i}\ \Gamma\left(\frac{q}{2}+i\right)\Gamma\left(\frac{v}{2}\right)}$$

$$\times \int_0^{c_1^*} t_1^{\frac{v+q}{2}+i-1}\, e^{\frac{-t_1}{2}}\, .\int_0^{c^*} \frac{t_2^{\frac{q}{2}+i-1}}{(1+t_2)^{\frac{v+q}{2}+i}}\, dt_2 . dt_1 \qquad \text{(A.3)}$$

where $c_1^* = \frac{m'\tau}{\sigma^2}$

Again changing the variables $g_1 = \frac{t_1}{2}$ and $g_2 = \frac{t_2}{(1+t_2)}$ so that equation (A.3) becomes

$$\iint_{R_1} f(u_1).f(u_2)du_1\, du_2 = \sum_{i=0}^{\infty} w_i(\lambda)\ \frac{1}{\Gamma\left(\frac{q}{2}+i\right)\Gamma\left(\frac{v}{2}\right)} \times \int_0^{\frac{c_1^*}{2}} g_1^{\frac{v+q}{2}+i-1}\, e^{-g_1}\, dg_1\, .\int_0^{\frac{c^*}{1+c^*}} g_2^{\frac{q}{2}+i-1}\, (1-g_2)^{\frac{v}{2}-1} dg_2 \qquad \text{(A.4)}$$

Now the integral parts in the above equation (A.4) may be written in terms of Incomplete Gamma and Beta function [see. Abramowitz and Stegun (1972)]

$$\iint_{R_1} f(u_1).f(u_2)du_1\, du_2$$

$$= \sum_{i=0}^{\infty} w_i(\lambda)\ \frac{1}{\Gamma\left(\frac{q}{2}+i\right)\Gamma\left(\frac{v}{2}\right)} \Gamma\left(\frac{v+q}{2}+i\right)\ G\left(\frac{v+q}{2}+i,\ \frac{c_1^*}{2}\right)\ B\left(\frac{q}{2}+i,\ \frac{v}{2}\right) \mathrm{I}_{\frac{qc}{v+qc}}\left(\frac{q}{2}+i, \frac{v}{2}\right)$$

Lastly, by putting the value of beta function, the above term may be written as

$$\iint_{R_1} f(u_1).f(u_2)du_1\, du_2 = \sum_{i=0}^{\infty} w_i(\lambda) G\left(\frac{v+q}{2}+i,\ \frac{c_1^*}{2}\right) \mathrm{I}_{\frac{qc}{v+qc}}\left(\frac{q}{2}+i, \frac{v}{2}\right) \qquad \text{(A.5)}$$

Similarly, on solving the second term in the equation (A.2) can be written as

$$\iint_{R_2} f(u_1).f(u_2)du_1\, du_2$$

$$= \sum_{i=0}^{\infty} w_i(\lambda)\ \frac{\Gamma\left(\frac{v+q}{2}+i\right)}{\Gamma\left(\frac{q}{2}+i\right)\Gamma\left(\frac{v}{2}\right)} . \int_{\frac{qc}{v+qc}}^{1} g^{\frac{q}{2}+i-1} (1-g)^{\frac{v}{2}-1}\, G\left(\frac{v+q}{2}+i,\ \ \frac{m\,\tau}{2\sigma^2(1-g)}\right) dg \qquad \text{(A.6)}$$

Now by substituting the value of (A.5) and (A.6) in (A.2), the distribution function of the Pre-test estimator has been obtained as described in Theorem 1.

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