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SOME SEQUENCE SPACES & ITS ALGEBRAIC PROPERTIES

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Abstract

The theory of statistical convergence has been discussed in trigonometric series, summability theory, measure theory, approximation theory, fuzzy set theory and so forth. The notion of limit is one of the central concept in mathematical analysis. The concept of ordinary limit has been generalized by mathematicians in various ways. One of the natural generalizations of limit is to define an operator extending the usual limit which assigns a value to some non-convergent sequences too. In this paper we introduce some new generalized difference double sequence spaces defined by Orlicz function and study different topological properties of these spaces and also establish some inclusion results among them.

Keywords: Sequence, Space, Properties etc.

1. Ideal Convergence

In recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces [1]. It has been presented a new generalization of statistical convergence and called it I- convergence. They used the notion of an ideal I of subsets of the set N to define such a concept [2].

Definition 1.1.: Let X 6= ∅. A class I of subsets of X is said to be an ideal in X provided:

(i) ∅∈ I;

(ii) A, B ∈ I implies A ∪ B ∈ I;

(iii) A ∈I , B ⊂ A implies B ∈ I.

I is called a non-trivial ideal if X ∉ I.

Definition 1.2: Let X 6= ∅. A non-empty class F of subsets of X is said to be a filter in X provided [3]:

(i) ∅∉ F;

(ii) A, B ∈ F implies A ∩ B ∈ F;

(iii) A ∈F , A ⊂ B implies B ∈ F.

If I is a nontrivial ideal in X, X 6= ∅ then the class.

F(I) = {M ⊂ X : (∃A ∈ I)(M = X \ A)} is a filter on X, called the filter associated with I.

Definition 1.3: A nontrivial ideal I in X is called admissible if {x} ∈ I for each x ∈ X [4].

Definition 1.4: A nontrivial ideal I is maximal if there cannot exist any nontrivial ideal J 6= I containing I as a subset.

Definition 1.5: A sequence (x_n) is said to be I−convergent to a number L ∈ R if for each ε > 0, {n ∈N : $|x_n - L| \geq \varepsilon$} ∈ I. The element L is called the I− limit of the sequence (x_n).

Definition 1.6: A sequence (x_n) is said to be I−null if L = 0.

Definition 1.7: A sequence (x_n) is said to be I−cauchy if for every ε > 0 there exist a number m = $m_0(\varepsilon)$ such that {n ∈ N : $|x_n - x_m| \geq \varepsilon$} ∈ I [5].

Definition 1.8: A sequence (x_n) is said to be I−bounded if there exists M > 0 such that {n ∈N : $|x_n| > M$} ∈ I [6].

2. REGULAR MATRIX TRANSFORMATION

The Silverman-Toeplitz theorem characterizes the regularity of two dimensional matrix transformation. It has given definitions for giving a value to a divergent double series by considering the double sequence for the series and established the conditions of regularity of linear transformations on double sequence spaces. Since then this concept have been studied by many researchers [7]. In 2004, it is presented an accessible multidimensional analogue of Brudno theorem using four dimensional matrix transformations of double sequences [8].

Let (a_{nk}) be a matrix defined by

$$a_{nk} = \begin{cases} 0, & k > n, \\ a_{nk}, & k \leq n. \end{cases}$$

For any given sequence (x_n) a new sequence (y_n) is defined as follows:

$$y_n = \sum_{k=1}^{n} a_{nk} x_k, \text{ for all } n \in N.$$

This transformation is said to be matrix transformation. If when (x_n) converges, (y_n) converges to the same value, then the transformation is said to be regular [9].

Theorem 2.1: A necessary and sufficient condition that the transformation of a triangular type matrix be regular is that

(a) $\lim_{n\to\infty} a_{n_k} = 0$, for every k;

(b) $\lim_{n\to\infty} \sum_{k=1}^{n} a_{nk} = 1$;

(c) $\sum_{k=1}^{n} |a_{nk}| < A$, for all n

Theorem 2.2: A necessary and sufficient condition that the transformation of a square type matrix be regular is that

(a) $\lim_{n\to\infty} a_{nk} = 0$, for every;

(b) $\sum_{k=1}^{\infty} |a_{nk}|$, converge for each n;

(c) $\sum_{k=1}^{\infty} |a_{nk}| < A$, for all n;

(d) $\lim_{n\to\infty} \sum_{k=1}^{\infty} a_{nk} = 1.$

3. SOME ALGEBRAIC PROPERTIES OF SEQUENCE SPACES

Here we discuss about some algebraic properties of sequence spaces that are relevant this work [10].

Definition 3.1: A sequence space E is said to be symmetric if $(x_{\pi(n)}) \in E$, whenever $(x_n) \in E$, where π is a permutation of N.

Definition 3.2: A sequence space E is said to be solid (or normal) if $(y_n) \in E$ whenever $(x_n) \in E$ and $|y_n| \leq |x_n|$ for all $n \in N$.

Definition 3.3: A sequence space E is said to be monotone if it contains the canonical preimages of all its step spaces [11].

Lemma 3.1: A sequence space E is normal implies that it is monotone.

Definition 3.4: A sequence space E is said to be convergence free if $(y_n) \in E$, whenever $(x_n) \in E$ and $x_n = 0$ implies $y_n = 0$.

Definition 3.5: A sequence space E is said to be sequence algebra if $(x_n * y_n) \in E$ whenever $(x_n), (y_n) \in E$.

4.TRIPLE SEQUENCE

A sequence is a function or a mapping from the set of natural numbers to the set of real numbers. A triple sequence (real or complex) can be defined as a function $x : N \times N \times N \longrightarrow R(C)$, where N, R and C denote the set of natural numbers, real numbers and complex numbers respectively. The different types of notions of triple sequences was introduced and investigated at the initial stage and many other researchers [12]. It has been introduced statistical convergence of triple sequences on probabilistic normed space. Later on, it has been introduced statistical convergence of triple sequences in topological groups [13].

Definition 4.1: A triple sequence (x_{lmn}) is said to be convergent to L in Pringsheim's sense if for every $\varepsilon > 0$, there exists $N(\varepsilon) \in N$ such that

$|x_{lmn} - L| < \varepsilon$, whenever $l \geq N, m \geq N, n \geq N$

and we write

$$\lim_{l,m,n \to \infty} x_{lmn} = L.$$

Note. A triple sequence is convergent in Pringsheim's sense may not be bounded.

Definition 4.2 : A triple sequence (x_{lmn}) is said to be Cauchy sequence if for every $\varepsilon > 0$, there exists $N(\varepsilon) \in N$ such that $|x_{lmn} - x_{pqr}| < \varepsilon$, whenever $l \geq p \geq N$, $m \geq q \geq N$, $n \geq r \geq N$, [14].

Definition 4.3 : A triple sequence (x_{lmn}) is said to be bounded if there exists $M > 0$, such that $|x_{lmn}| < M$, for all $l, m, n \in N$.

A subset E of $N \times N \times N$ is said to have density $\delta(E)$ if the limit given by

$$\delta(E) = \lim_{l,m,n \to \infty} \frac{1}{lmn} \Sigma_{p \leq l} \Sigma_{q \leq m} \Sigma_{r \leq n} \chi_E(p,q,r) \text{ exist.}$$

Thus a triple sequence (x_{lmn}) is said to be statistically convergent to L in Pringshiem's sense if for every $\varepsilon > 0$, $\delta(\{(l, m, n) \in N \times N \times N : |x_{lmn} - L| \geq \varepsilon\}) = 0$. We write this by

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$$\text{stat} - \lim_{l,m,n\to\infty} x_{lmn} = L.$$

Statistically convergent sequences were introduced and investigated and it was investigated by others also [15].

Definition 4.4: A triple sequence (x_{lmn}) is said to be a statistically cauchy sequence if for every $\varepsilon > 0$, there exists $p = p(\varepsilon)$, $q = q(\varepsilon)$ and $r = r(\varepsilon)$, such that $\delta(\{(p, q, r) \in N \times N \times N : |x_{lmn} - x_{pqr}| \geq \varepsilon\}) = 0$.

Definition 4.5 : A triple sequence (x_{lmn}) is said to be statistically bounded if there exists $M > 0$ such that $\delta(\{(l, m, n) \in N \times N \times N : |x_{lmn}| > M\}) = 0$, for all $l, m, n \in N$.

Definition 4.6: A triple sequence (x_{lmn}) is said to be statistically convergent to L in Pringshiem's sense if for every $\varepsilon > 0$, $\delta(\{(l, m, n) \in N \times N \times N : |x_{lmn} - L| \geq \varepsilon\}) = 0$. We write this by, $\text{stat} - l_{iml,m,n}\to\infty x_{lmn} = L$.

Definition 4.7: A triple sequence (x_{lmn}) is said to be I- convergence to a number L if for every $\varepsilon > 0$, $\{(l, m, n) \in N \times N \times N : |x_{lmn} - L|) \geq \varepsilon\} \in I$, in this case we write $I - \lim x_{lmn} = L$.

Definition 4.8: A triple sequence (x_{pqr}) is said to be I-null if $L = 0$. In this case we write $I - \lim x_{pqr} = 0$.

Definition 4.9 : A triple sequence space E is said to be solid (or normal) if $(\alpha_{lmn}x_{lmn}) \in E$ whenever $(x_{lmn}) \in E$ and for all sequences (α_{lmn}) of scalars with $|\alpha_{lmn}| \leq 1$, for all $l, m, n \in N$.

Definition 4.10 : A triple sequence space E is said to be monotone if it contains the canonical preimages of all its step spaces.

Remark 4.1: A triple sequence space is solid implies that it is monotone.

Definition 4.11: A triple sequence space E is said to be convergence free if $(y_{lmn}) \in E$, whenever $(x_{lmn}) \in E$ and $x_{lmn} = 0$ implies $y_{lmn} = 0$.

Definition 4.12 : A triple sequence space E is said to be sequence algebra if $(x_{lmn} * y_{lmn}) \in E$, whenever $(x_{lmn}) \in E$ and $y_{lmn} \in E$.

5. CONCLUSION

It is concluded that So far as we know in several branches of analysis, for instants, the structural theory of topological vector spaces, shauder basis theory, summability theory and theory of functions, the study of sequence spaces occupies a very prominent position. The impact and importance of this study can be appreciated when one sees the construction of numerous examples locally convex spaces obtained as a consequence of the dual structure displayed by several pairs of distinct sequence spaces, thus reflecting in depth the distinguishing structural features of the spaces in question. Besides these distinct sequence spaces endowed with different polar topologies provide an excellent source to vector space pathalogists for the introduction on locally convex spaces to several new and penetrating notions implicit in the theory of Banach spaces. Apart from this the theory of sequence spaces is a powerful tool for obtaining positive results concerning shauder basis and their associated types. Moreover, the theory of sequence spaces has made remarkable advances in enveloping summability theory via unified techniques effecting matrix transformation from one sequence space into another.

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